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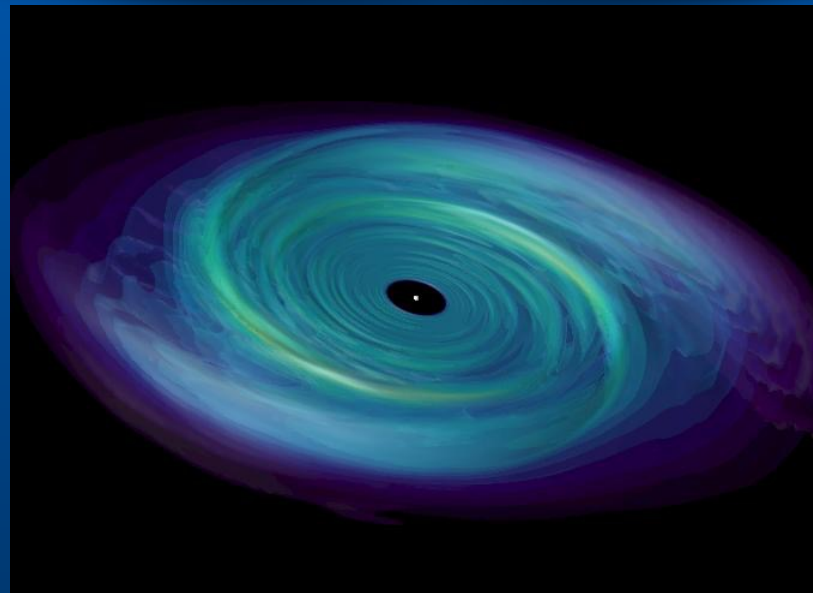
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# Black hole astrophysics



Gustavo E. Romero

Grupo de Astrofísica Relativista y Radioastronomía

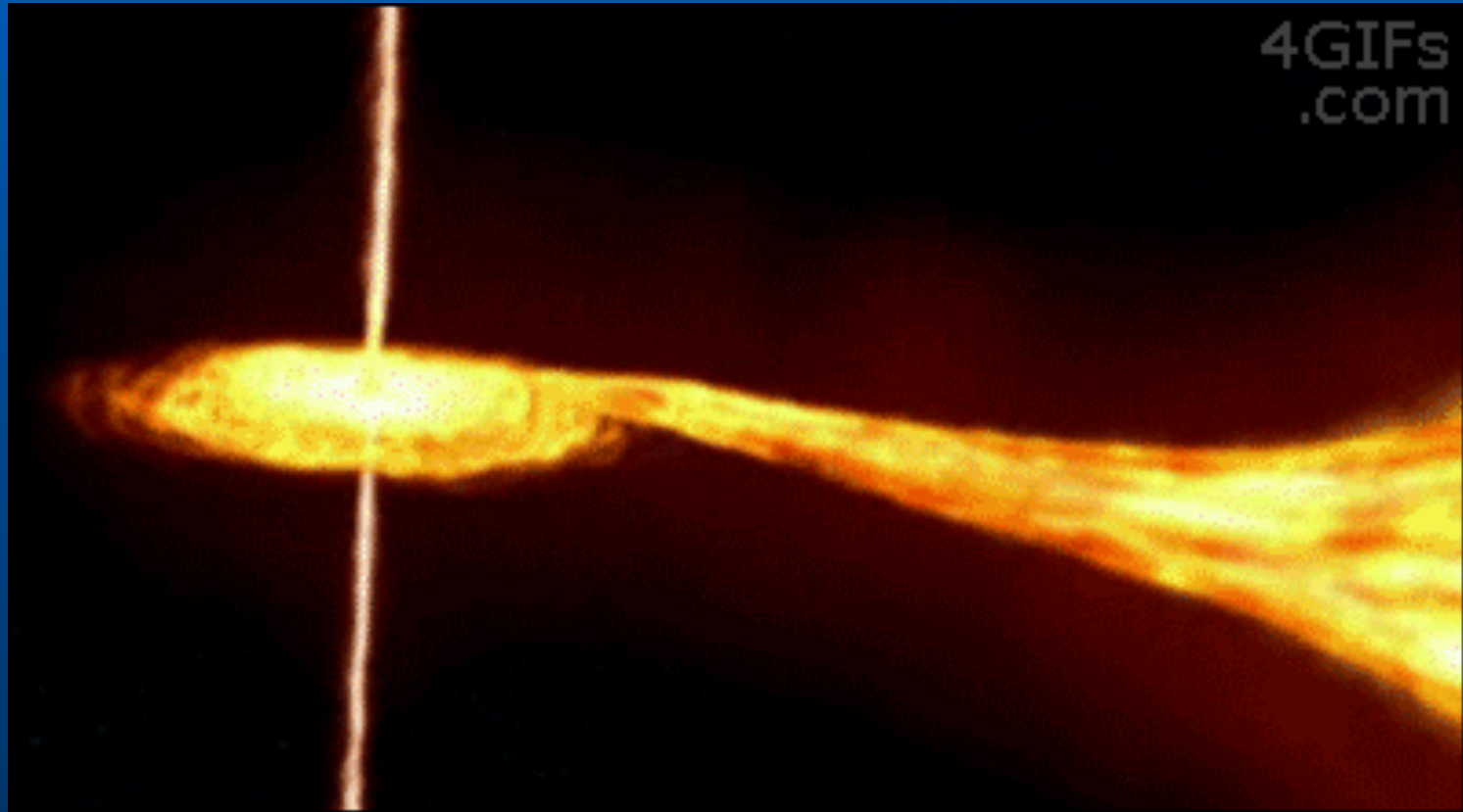
Instituto Argentino de Radioastronomía, CONICET-CIC-UNLP

Facultad de Ciencias Astronómicas y Geofísicas, UNLP

romero@fcaglp.unlp.edu.ar

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## Feeding black holes: microquasars

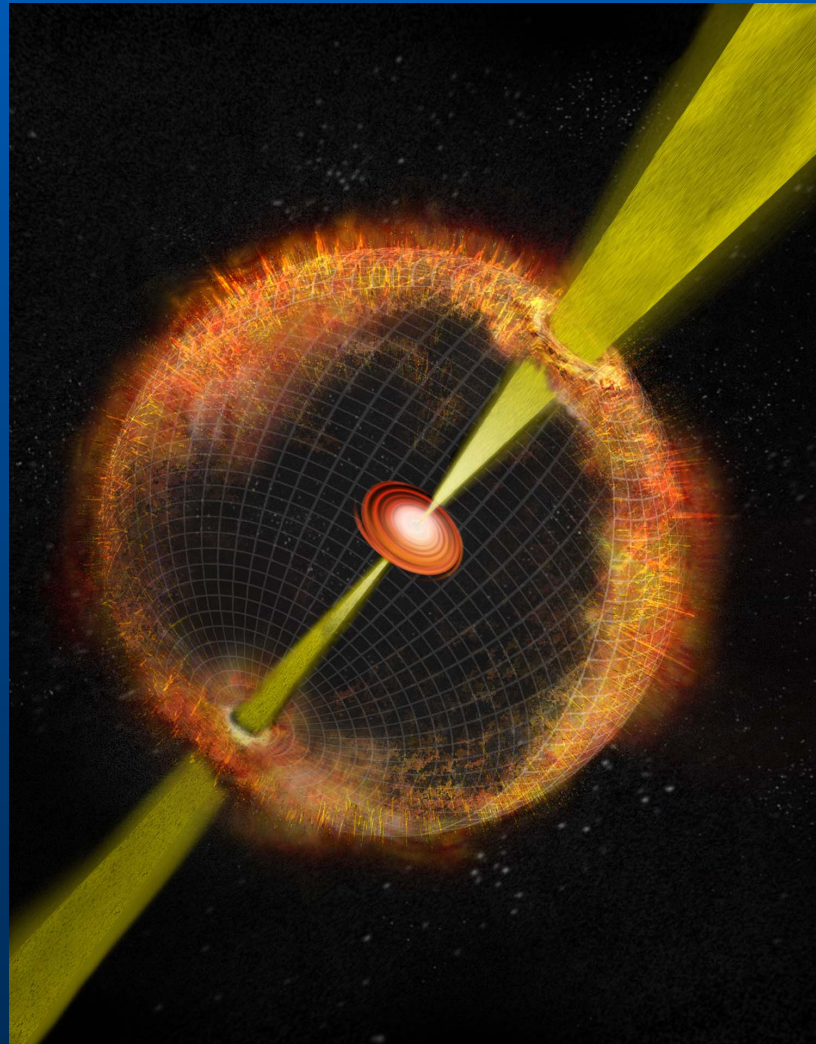




# Feeding black holes: Active Galactic Nuclei



# Feeding black holes: Gamma-Ray Bursts



# Feeding black holes: Gamma-Ray Bursts



# Feeding isolated black holes



## What is accretion?

Accretion is the process of matter falling into the potential well of a gravitating object. The accretion of matter with no angular momentum is basically determined by the relation between the speed of sound  $a_s$  in the matter and the relative velocity  $v_{\text{rel}}$  between the accretor and the medium.

The accretion of matter with angular momentum can lead to the formation of an accretion disk around the compact object.

## Types of accretion

There are four basic regimes of accretion onto a black hole:

- Spherical symmetric accretion. It occurs when  $v_{\text{rel}} \ll a_s$  and the accreting matter does not have any significant angular momentum.
- Cylindrical accretion. The angular momentum of the medium remains small but now  $v_{\text{rel}} \geq a_s$ .
- Disk accretion. The total angular momentum of matter is enough as to form an accretion disk around the black hole.
- Two-stream accretion. Both a quasi-spherically symmetric inflow of matter coexists with disk accretion (e.g. Narayan & Yi 1994).



The hydrodynamic description of accretion (or any other physical process) is valid if the mean free path of the particles in the medium is shorter than the typical size scale of the system. In the case of accretion the self-gravitation of the fluid is usually negligible, so the characteristic length scale is, as we shall see, the *gravitational capture radius* or *accretion radius*,  $R_G$  or  $R_{\text{accr}}$ .

This quantity is roughly equal to the distance to the accretor at which the kinetic energy of an element of matter is of the order of its gravitational energy,

$$\frac{1}{2}(a_s^2 + v_{\text{rel}}^2) = \frac{GM}{R_{\text{accr}}}.$$



## Basic equations

$$R_G = \frac{2GM}{a_s^2 + v_{\text{rel}}^2}.$$

In the absence of radiation, the equations that fully describe the accretion process are:

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v} = -\frac{1}{\rho} \vec{\nabla} P - \vec{\nabla} \phi,$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0,$$

$$P = P(\rho),$$

$$\nabla^2 \phi = -4\pi G \rho_{\text{tot}}.$$

## Gravitational capture

A key parameter in the study of accretion flows is the mass accretion rate  $dM/dt$ , defined as the mass per unit time captured by the gravitating center.

$$\dot{M} = \sigma_G \rho v_{\text{rel}}$$

This cross section depends strongly on the nature of the gas. If the gas is made of collisionless non-relativistic particles, the gravitational capture cross section in a Schwarzschild black hole is

$$\sigma_{G(\text{collisionless})} = 4\pi \left( \frac{c}{v_{\infty}} \right)^2 R_{\text{Schw}}^2,$$

## Gravitational capture

$$\sigma_{\text{G(fluid)}} \approx \pi R_{\text{G}}^2$$

Then

$$\frac{\sigma_{\text{G(collisionless)}}}{\sigma_{\text{G(fluid)}}} \propto \left(\frac{c}{v_{\infty}}\right)^2 \left(\frac{R_{\text{Schw}}}{R_{\text{G}}}\right)^2 \ll 1.$$

Under typical conditions in the interstellar medium the capture cross section for accretion of a fluid is about a million times that for the accretion of collisionless particles.

## Spherically symmetric accretion

Equation of state:

$$P \propto \rho^\gamma.$$

From the Euler equation:

$$\frac{v^2}{2} + \frac{\gamma}{\gamma - 1} \frac{P}{\rho} - \frac{GM}{R} = \text{constant} = \epsilon_0.$$

The continuity equation can be written as:

$$\dot{M} = 4\pi R^2 \rho v = \text{constant},$$

where we have considered accretion over a sphere of radius  $R$ .

The boundary conditions at infinity imply:

$$\epsilon_0 = \frac{\gamma}{\gamma - 1} \frac{P_\infty}{\rho_\infty} = \frac{a_\infty^2}{\gamma - 1}.$$

The speed of sound is

$$a_s = \sqrt{\frac{\partial P}{\partial \rho}} = \sqrt{\frac{\gamma P}{\rho}}.$$

Then,

$$\frac{v^2}{2} + \frac{a_s^2}{\gamma - 1} = \frac{GM}{R} + \frac{a_\infty^2}{\gamma - 1},$$

and

$$\dot{M} = 4\pi r^2 v \rho,$$

$$v = \frac{\dot{M}}{4\pi \rho_\infty R^2} \left( \frac{a_\infty}{a_s} \right)^{2/(\gamma-1)}.$$

$$a_s^2 = \gamma \frac{P}{\rho} = a_\infty^2 \left( \frac{\rho}{\rho_\infty} \right)^{\gamma-1},$$

There is a critical point at  $R = R_s$  which the gas velocity overcomes the speed of the sound. At  $R \ll R_s$  the matter is practically in a state of free fall toward the black hole. This is because the flow becomes supersonic and then the underlying layers do not affect the entrained matter. Then, we can write:

$$v \approx \sqrt{\frac{2GM}{R}}, \quad \dot{M} = 4\pi r^2 v \rho,$$

and

$$\rho \approx \frac{\dot{M}}{4\pi\sqrt{2GM}} R^{-3/2}.$$

If the effect of radiation is to be taken into account, then we must add the second law of thermodynamics. The variation of the internal energy per unit mass of the gas is

$$de = dQ - PdV,$$

where  $V = 1/\rho$  is the specific volume and  $dQ$  is the heat exchanged per unit mass.

For a monoatomic gas

$$P = \frac{1}{V} \frac{m}{\mu m_u} kT = \rho \frac{k}{\mu m_u} T$$

$$E = \frac{3}{2} k_B T$$

Energy associated with one atom

$$\frac{3k}{2\mu m_p} \frac{dT}{dt} = \frac{k}{\mu m_p} \frac{T}{\rho} \frac{d\rho}{dt} - \alpha_{\text{ff}} T^{1/2} \rho + \frac{dQ}{dt}.$$

$$(\alpha_{\text{ff}} \approx 5 \times 10^{20} \text{ erg g}^{-1} \text{ s}^{-1})$$



Using  $dr = v dt$ , we can obtain the equation for the temperature distribution in a steady-state spherically symmetric accretion flow:

$$\frac{dT}{dr} = -\frac{T}{r} - \alpha_{\text{ff}} \rho(r_s) \left( \frac{r_s}{2GM} \right)^{1/2} \frac{T^{1/2}}{r} + \frac{2\mu m_p}{3k} \frac{dQ}{dr}.$$

If there are no additional radiation losses besides free-free radiation

$$T = \left[ K \ln \left( \frac{r}{R_G} \right) + T_\infty^{1/2} \right]^2,$$

where we have assumed that at  $R = R_G$  the temperature is  $T_\infty$ .

The previous equation shows that under such conditions the temperature *decreases* as the flow approaches the black hole. A flow that behaves in this way is called a *cooling flow*.

The radial free fall time is

$$t_{\text{ff}} \approx \frac{r}{v_{\text{ff}}} \propto r^{3/2},$$

whereas the cooling time for Bremsstrahlung losses ( $dQ/dT \propto T^{1/2}\rho$ ) is<sup>3</sup>

$$t_{\text{cool,Br}} \approx \frac{3kT/2\mu m_p}{\alpha_{\text{ff}} T^{1/2} \rho} \propto \frac{\sqrt{T}}{\rho} \approx r.$$

Comparing both timescales we see that the relative role of cooling decreases as the black hole is approached.

## Eddington luminosity

Close to the black hole there could be sources of radiation, for example if a magnetic field and dissipation of angular momentum are involved. The outgoing radiation will pass through the accretion flow and may influence its dynamics. Each particle experiences a force

$$F_{\text{rad}} = \frac{\sigma L}{4\pi r^2 c},$$

$$\sigma = \sigma_{\text{T}} \approx 0.66 \times 10^{-24} \text{ cm}^2.$$

$$F_{\text{grav}} = \frac{GMm_p}{r^2}.$$

## Eddington luminosity

Then, if the luminosity equals

$$L_{\text{Edd}} \equiv \frac{4\pi G M m_p c}{\sigma}$$

the gravitational force and the radiation force are balanced and spherical accretion is stopped. This critical luminosity is called the *Eddington luminosity* of the accreting source. In the case of Thomson scattering ( $\sigma = \sigma_T$ ) it takes the value

$$L_{\text{Edd}} \approx 1.3 \times 10^{38} \left( \frac{M}{M_\odot} \right) \text{ erg s}^{-1}.$$

## Eddington luminosity

Associated with the critical luminosity we can define the *Eddington accretion rate* as

$$\dot{M}_{\text{Edd}} = \frac{L_{\text{Edd}}}{c^2} \approx 0.2 \times 10^{-8} \left( \frac{M}{M_{\odot}} \right) M_{\odot} \text{ yr}^{-1}.$$

The *Eddington temperature*  $T_{\text{Edd}}$  is the characteristic temperature of a blackbody of radius equal to the Schwarzschild radius that radiates at  $L = L_{\text{Edd}}$ ,

$$T_{\text{Edd}} = \left( \frac{L_{\text{Edd}}}{4\pi\sigma_{\text{SB}}R_{\text{Schw}}^2} \right)^{1/4} \approx 6.6 \times 10^7 \left( \frac{M}{M_{\odot}} \right)^{-1/4} \text{ K}.$$

## Wind pressure

Another way to inhibit spherical accretion is the production of winds or particle ejection in the inner accreting regions. If  $L_{ej}$  is the power carried away by the ejected particles and  $v_{ej}$  is their velocity, the exerted pressure will be:

$$P_{ej} = \frac{L_{ej}}{4\pi R^2 v_{ej}}.$$

If the central source ejects particles before the onset of the spherical accretion, pressures must be equated at the gravitational capture radius to find the critical luminosity in ejected particles:

$$\frac{L_{ej}}{4\pi r_{\text{accr}}^2 v_{ej}} \approx \rho_{\infty} a_{\infty}^2 \approx \frac{\dot{M} a_{\infty}}{\pi r_{\text{accr}}^2}.$$

From here we get:

$$L_{ej}^{\text{crit}} \approx 4\dot{M} a_{\infty} v_{ej}.$$

Using the fact that a fraction  $\eta$  of the accretion power is released as radiation, i.e.

$$L = \eta \dot{M} c^2,$$

we obtain

$$L_{ej}^{\text{crit}} \approx 4 \frac{L}{\eta} \left( \frac{a_{\infty} v_{ej}}{c^2} \right).$$

It can be seen that a weak wind can stop the the spherical accretion.

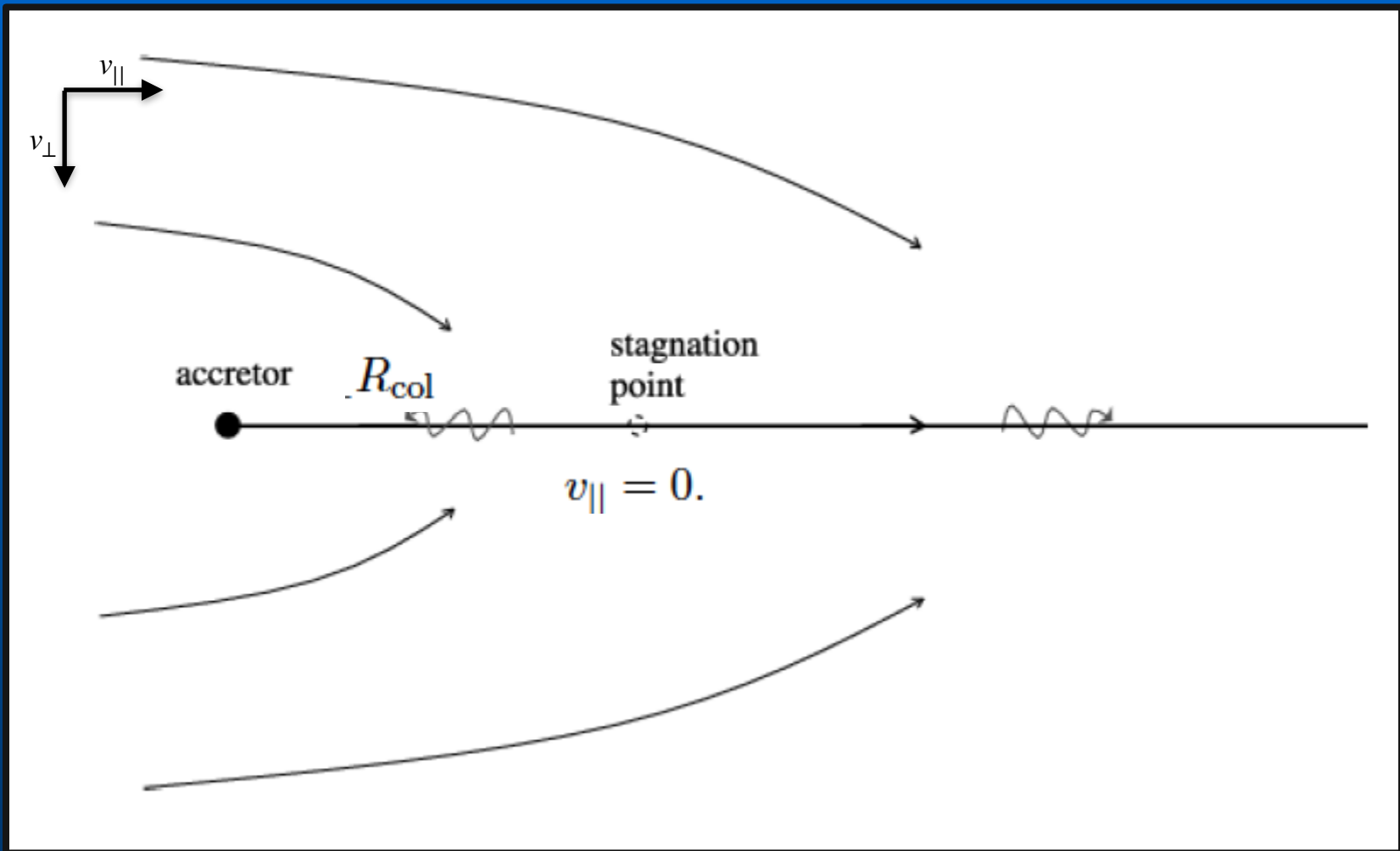
## Cylindrical accretion

The problem of cylindrical accretion is the problem of the determination of the gas accretion onto a moving gravitating center. Unlike the case of spherical accretion, the problem is quite complex and there are not analytical solutions of it.

In cylindrical accretion, the velocity of the compact object with respect to the medium  $v_{\text{rel}}$  is not negligible.

The symmetry axis of the problem is determined by the line of motion of the object. The stream lines of the fluid then are hyperbolae centered on this axis. As individual particles move, the angular momentum is conserved relative to the accreting object.





$$|R \times v| = bv_{\text{rel}},$$

$$v_{\perp} R_{\text{col}} = bv_{\text{rel}}.$$

The particles to be captured by the compact object are those for which the velocity is lower than the parabolic velocity:

$$v_{\parallel} \leq \sqrt{\left(\frac{2GM}{R_{\text{col}}}\right)}.$$

The conservation of energy implies:

$$\frac{1}{2}(v_{\parallel}^2 + v_{\perp}^2) - \frac{GM}{R_{\text{col}}} = \frac{1}{2}v_{\text{rel}}^2.$$

Only particles satisfying  $v_{\perp} \leq v_{\text{rel}}$  will be captured. The gravitational capture cross section is determined by the capture radius:

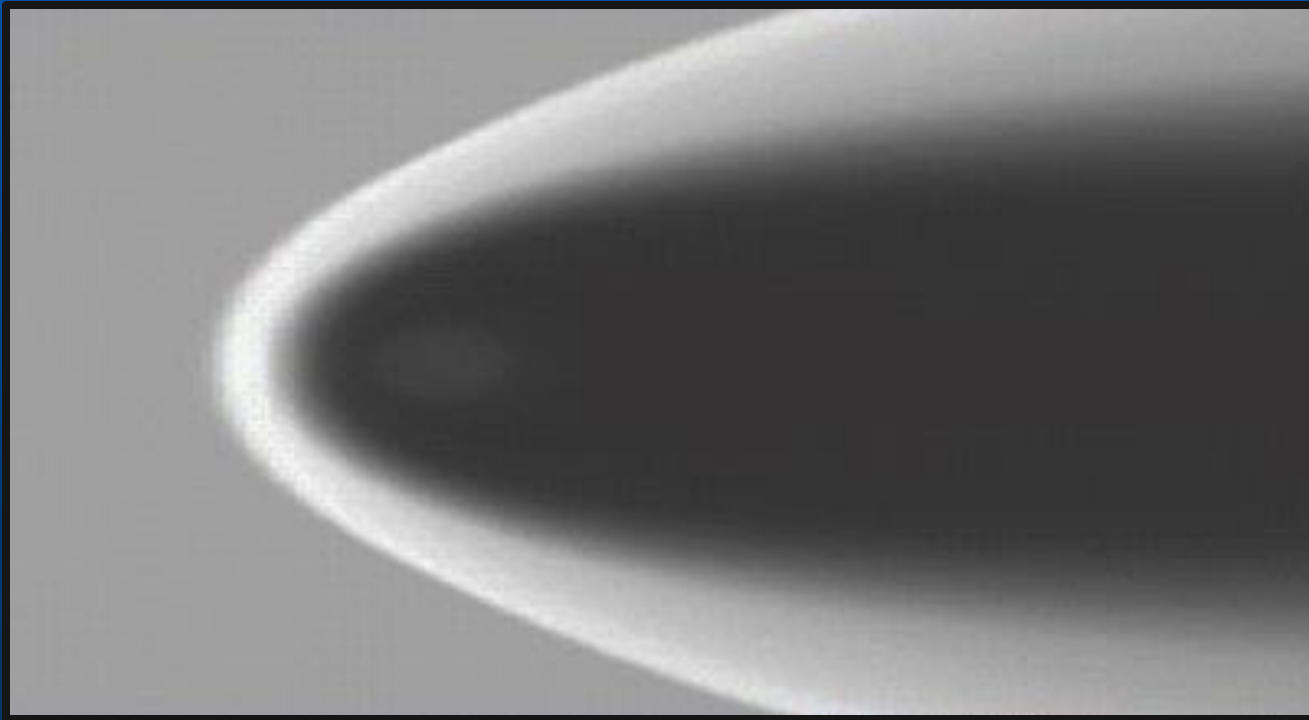
$$\sigma_{\text{G}} \approx \pi R_{\text{G}}^2.$$

$$R_{\text{G}} = \frac{2GM}{a_{\text{s}}^2 + v_{\text{rel}}^2}.$$

$$R_{\text{HL}} = \frac{2GM}{v_{\infty}^2} \longrightarrow \dot{M}_{\text{HL}} = \pi R_{\text{HL}}^2 \rho_{\infty} v_{\infty} = \frac{4\pi G^2 M^2 \rho_{\infty}}{v_{\infty}^3}$$

$$\dot{M} \approx \frac{2\pi G^2 M^2 \rho_{\infty}}{(a_{\infty}^2 + v_{\infty}^2)^{3/2}}.$$

If we work in the fluid approximation, the supersonic motion of the compact object will lead to the formation of a bow shock



A moving body through a gaseous medium with produce density perturbations. If these perturbations are small, we can write:

$$\rho = \rho_{\infty} + \delta\rho, \quad \delta = \frac{\delta\rho}{\rho_{\infty}}.$$

In addition, we assume that the gravitational center is point-like and moves at  $v_{\infty}$ . The accretion rate is:

$$\dot{M} = \xi_1 \pi R_G^2 v_{\infty} \rho_{\infty},$$

wheres  $\xi_1$  is a dimensionless parameter of the order of unity.

Perturbing and linearizing:

$$\frac{\partial v}{\partial t} = -a_\infty^2 \vec{\nabla} \delta + \vec{\nabla} \phi,$$

$$\frac{\partial \delta}{\partial t} + \vec{\nabla} v = -\xi_1 \pi R_G^2 v_\infty \delta(R - v_\infty t),$$

$$\nabla^2 \delta \phi = -4\pi G(\rho_{\text{tot}} + \delta \rho_\infty).$$

The Jeans wavelength is defined such that any small sinusoidal density disturbance with a wavelength exceeding  $2\pi/k$ , will be gravitationally unstable. The Jeans critical mass is usually defined as the density times the cubic of the length. Higher masses than the Jeans mass start to condense gravitationally.

$$k_J^2 = \frac{4\pi G \rho_\infty}{a_\infty^2},$$

Using the Jeans wavelength:

$$k_J^2 = \frac{4\pi G \rho_\infty}{a_\infty^2},$$

We get:

$$\square^2 \delta + k_J^2 a_\infty^2 \delta = -4\pi G \rho_{\text{tot}} + \xi_1 \pi R_G^2 v_\infty \frac{\partial}{\partial t} \delta(R - v_\infty t).$$

To simplify, we center the coordinate system in the moving object and neglect self-gravity of the gas ( $k_J$ ):

$$(a_\infty^2 - v_\infty^2) \nabla^2 \delta = 4\pi G M \delta(R).$$



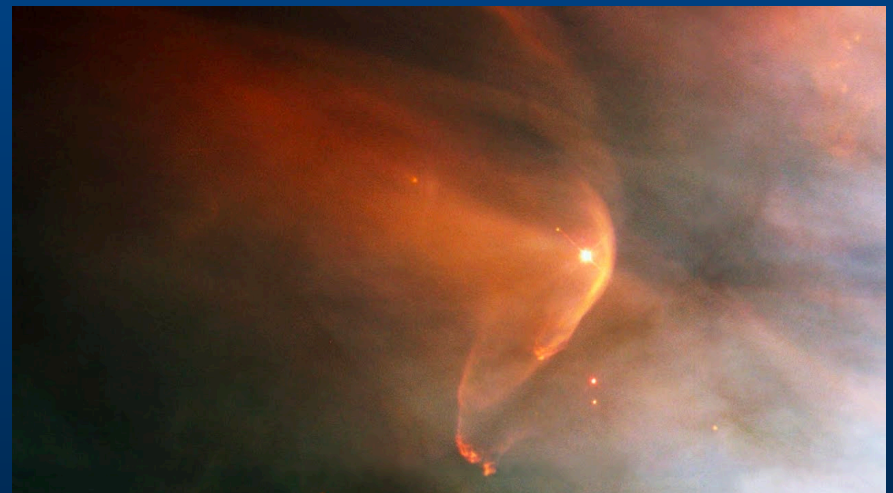
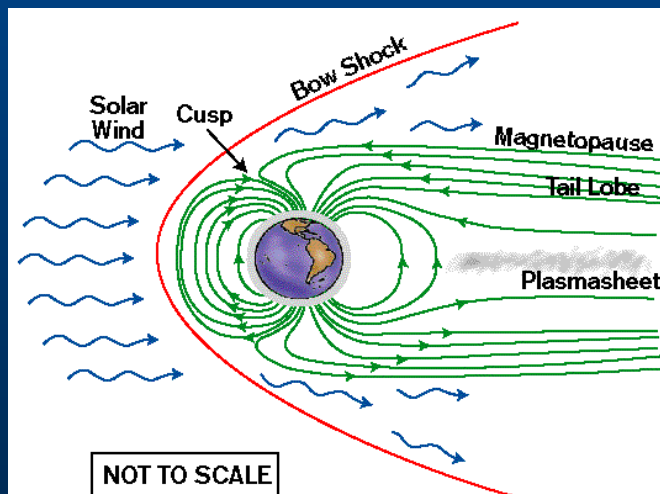
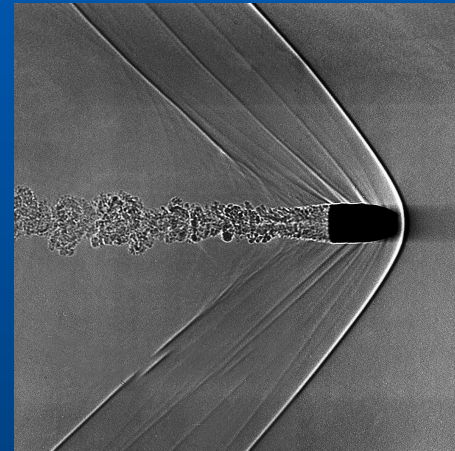
The solution is (Lipunov 1992):

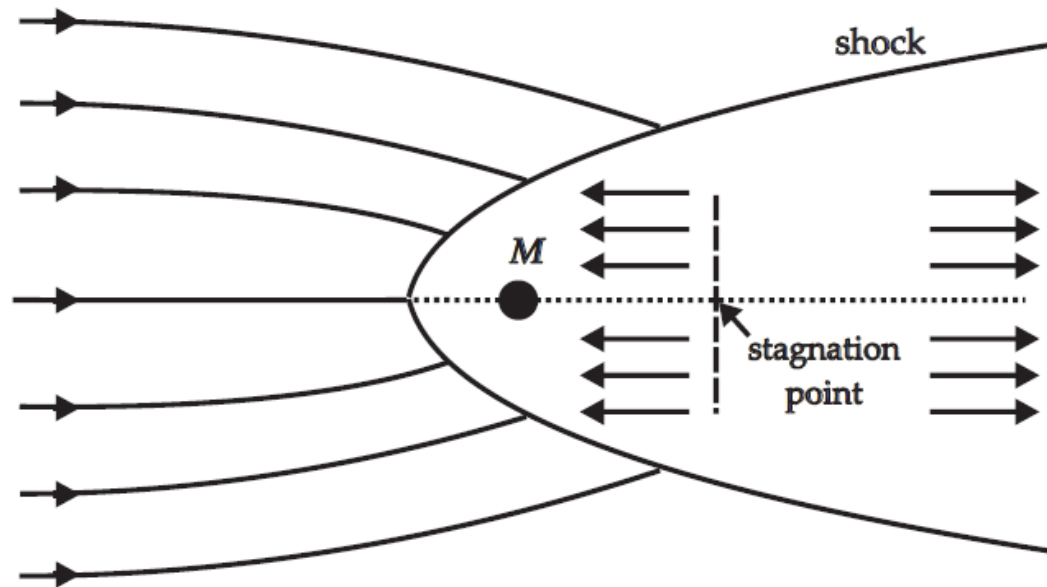
$$\delta = \frac{R_G v_\infty}{a_\infty R \left(1 - \frac{v_\infty^2}{a_\infty^2 \sin^2 \theta_{\text{sh}}}\right)^{1/2}}.$$

If we introduce the Mach number  $M_M = v_\infty/a_\infty$ , we see that there is a singularity at the surface of the cone described by:

$$\sin \theta_{\text{sh}} = \frac{1}{M_M}.$$

The solution is not valid close to the cone, which implies that a shock wave is formed, with a form of a cone with opening angle  $\theta_{sh}$ . This shock is called a “bow shock”.



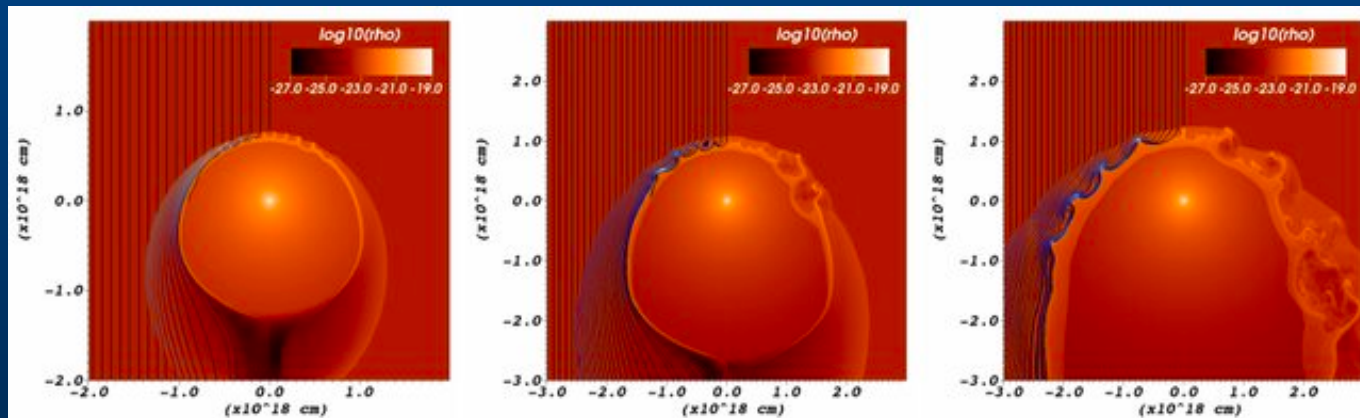


**Fig. 4.3** Sketch of the geometry of an axisymmetric accretion flow in the Bondi-Hoyle model. The arrows indicate the direction of the flow.

A force due to dynamical friction opposes to the motion of the compact object, slowing it down. Such a force is the result that the density in the background matter in the wake is higher than in front of the moving center. The dynamic friction force is:

$$F_{\text{fr}} = \pi R_G^2 \rho_\infty v_\infty.$$

Numerical simulations show that the accretion flow pattern is complicated and dependent on the efficiency of the gas cooling mechanism. The simulations also show the formation of a frontal shock at a distance  $\sim R_G$ . This region is prone to suffer Rayleigh-Taylor instabilities.



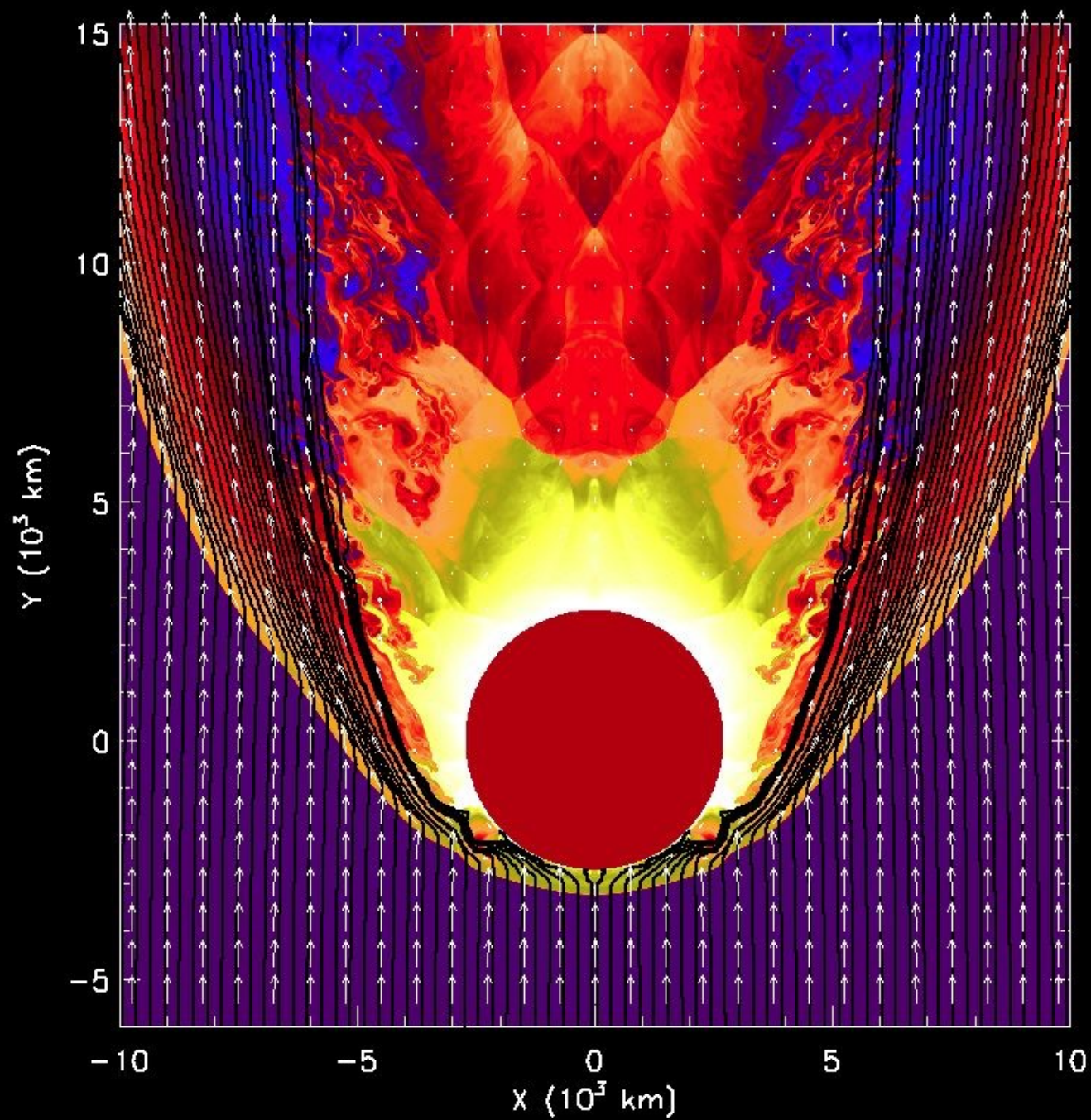
The temperature in the wake of the the shock is (Lang 1999):

$$T_{\text{sh}} = \frac{m_p v_\infty^2}{6k} \approx 2.5 \times 10^5 \left( \frac{v_\infty}{10^7 \text{ cm s}^{-1}} \right)^2 \text{ K},$$

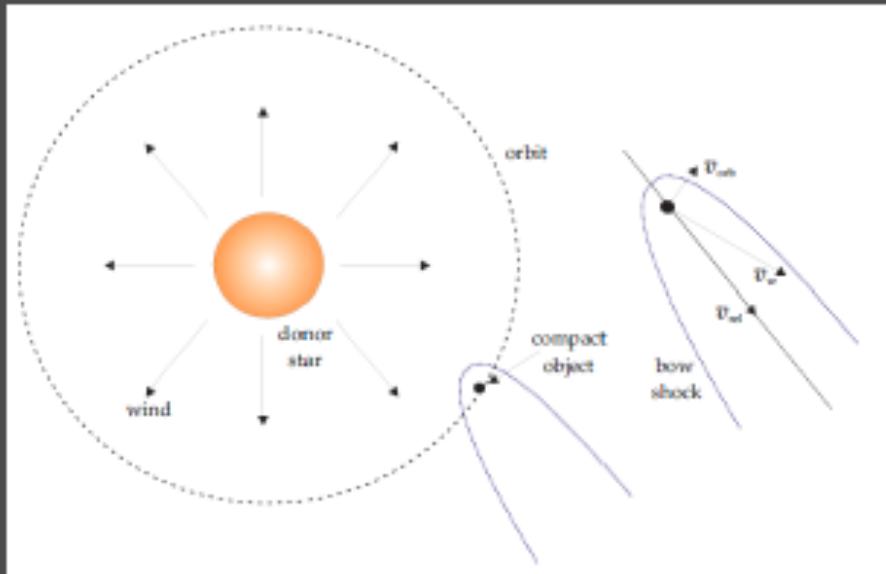
where  $k$  is the Boltzmann's constant.

A realistic study of the accretion regimes onto moving objects requires extensive numerical simulations.





# Black hole in a massive binary system

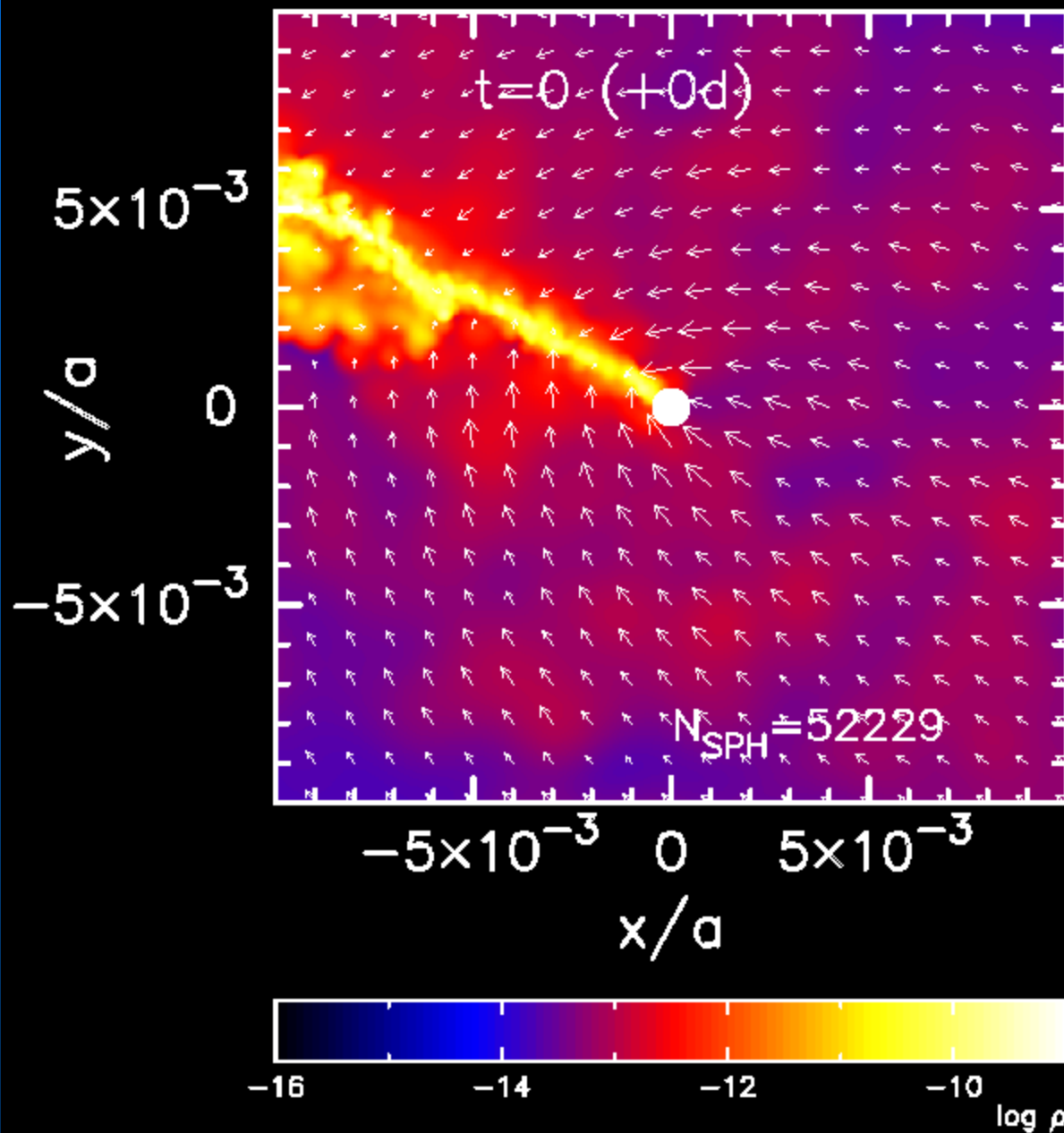


$$R_{\text{accr}} \approx \frac{2GM_c}{v_{\text{rel}}^2} \quad v_{\text{rel}}^2 \approx v_c^2 + v_w^2$$

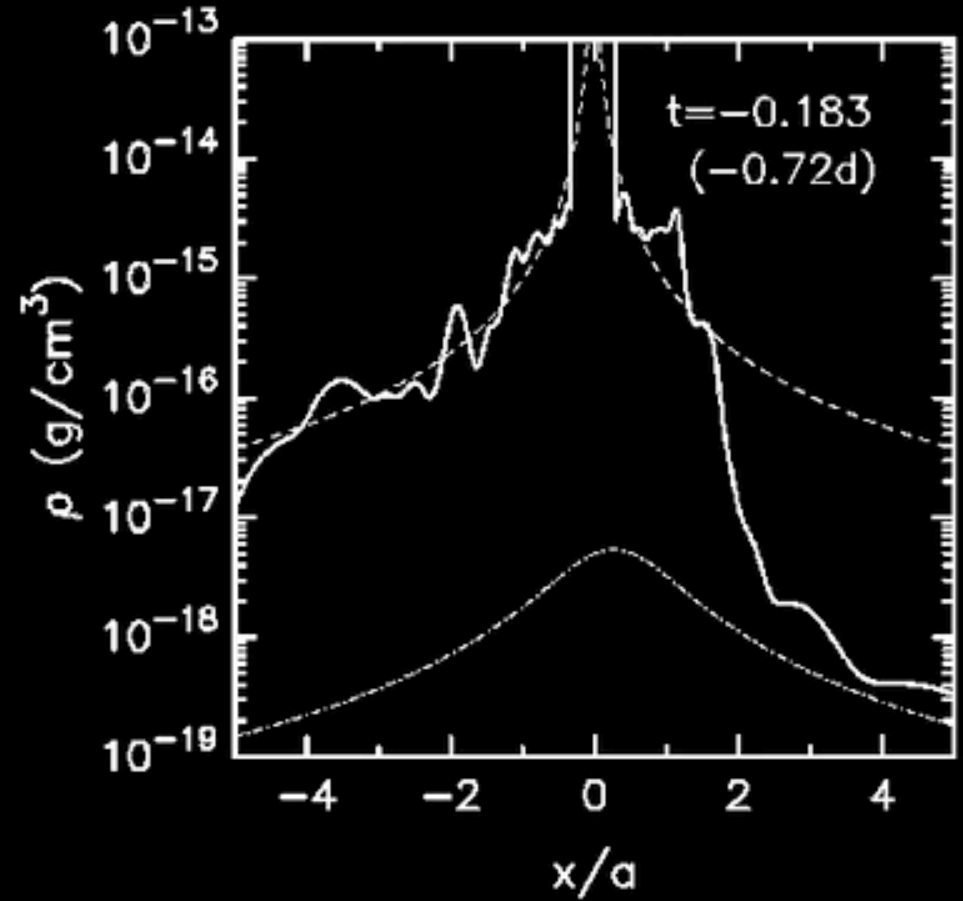
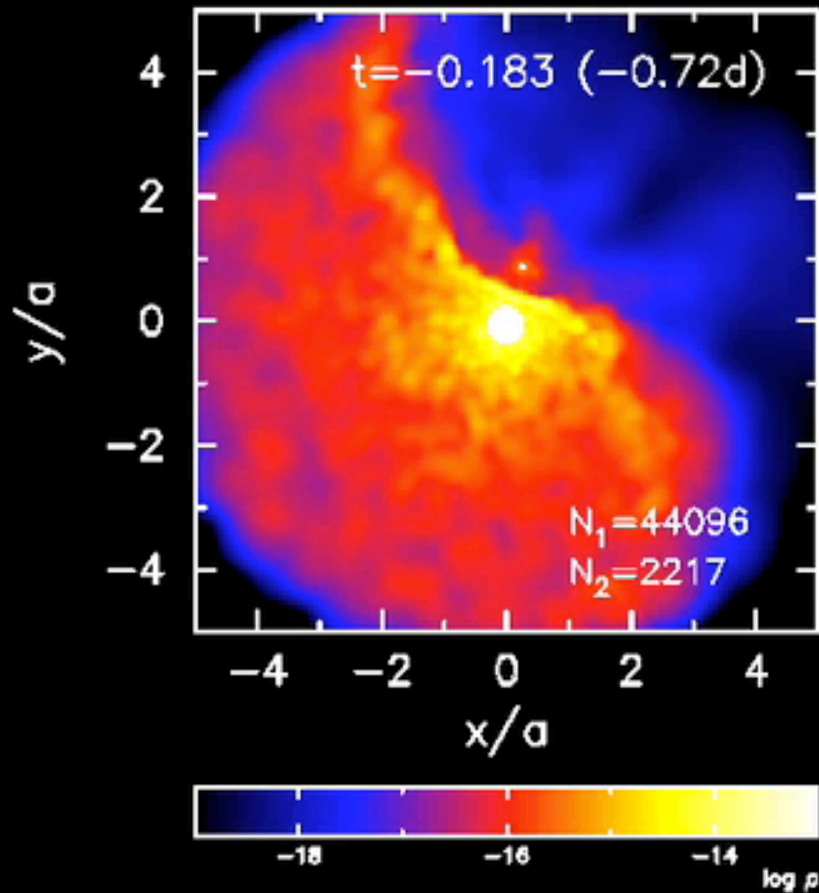
$$\dot{M} \approx \pi R_{\text{accr}}^2 \rho_w v_{\text{rel}} = 4\pi G^2 M_c^2 \rho_w v_{\text{rel}}^{-3}$$

¿Es eficiente?

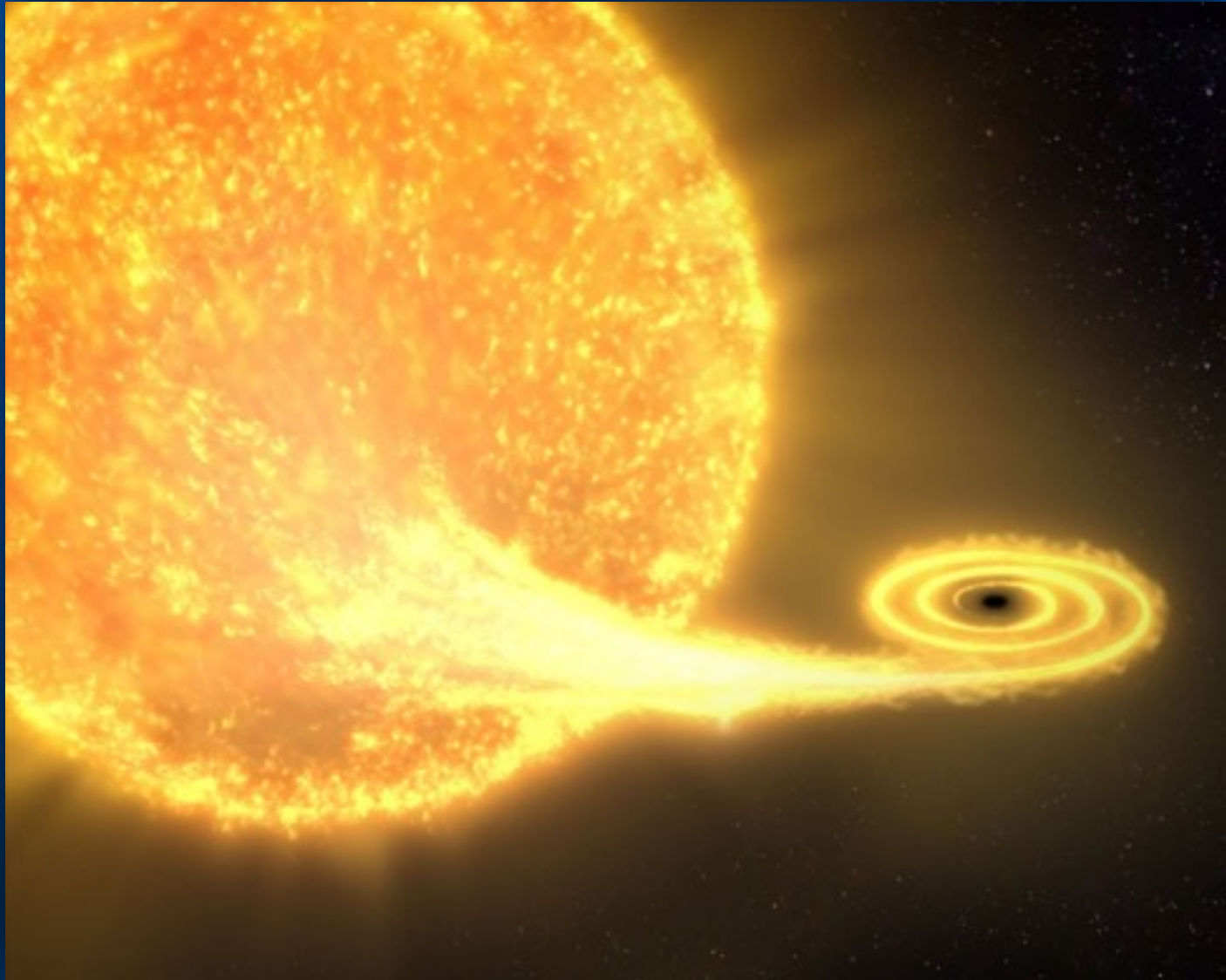
$$\frac{\dot{M}}{\dot{M}_*} \approx \frac{\pi R_{\text{accr}}^2 \rho_w v_{\text{rel}}}{4\pi R_{\text{orb}}^2 \rho_w v_w} = \frac{1}{4} \left( \frac{R_{\text{accr}}}{R_{\text{orb}}} \right)^2 \left( \frac{v_{\text{rel}}}{v_w} \right) \approx 10^{-2} - 10^{-4}$$





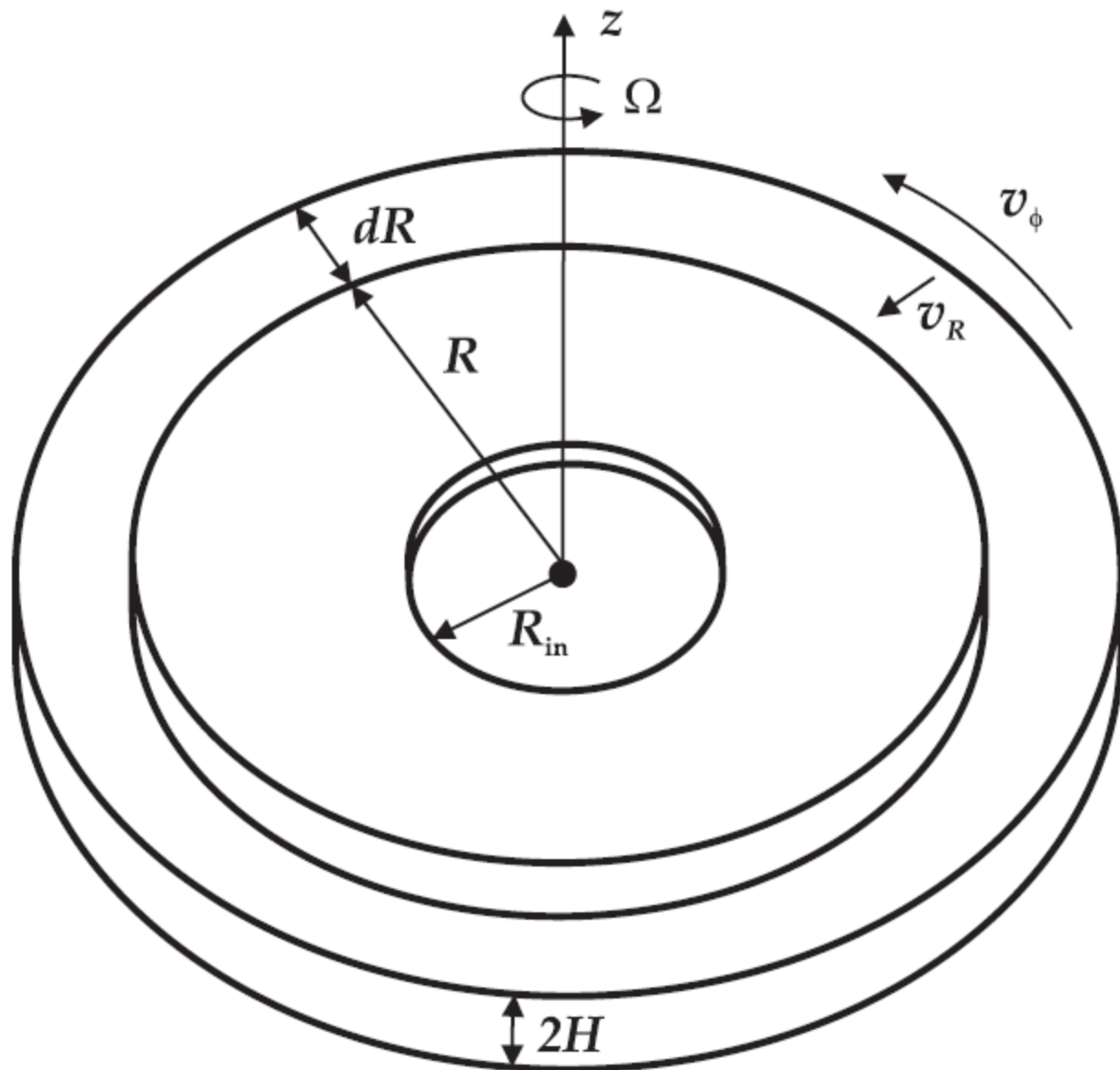


# Disk accretion



## Disk accretion

In most realistic astrophysical situations the matter captured by a gravitational field will have a total non-zero angular momentum. The accretion of matter with angular momentum onto a black hole leads to the formation of an accretion disk. The main difficulty in the formulation of a consistent theory of accretion disks lies in the lack of knowledge on the nature of turbulence in the disk and, therefore, in the estimate of the dynamic viscosity.



## Disk accretion: thin disks

We shall start with the following simplifying assumptions:

- 1) the disk is thin, i.e. its characteristic scale in the  $z$ -axis is  $H \ll R$ ,
- 2) the matter in the disk is in hydrostatic equilibrium in the  $z$ -axis,
- 3) self-gravitation of the disk can be neglected.

Condition 2) can be expressed as:

$$\frac{1}{\rho} \frac{dP}{dz} = -\frac{GM}{R^3} z.$$

## Disk accretion

If  $a_s$  is the sound speed,  $H = \Delta z$  is the half-thickness of the disk, and  $P = \rho a_s^2$ , we can re-write:

$$a_s = \omega_K H,$$

$$\omega_K = \sqrt{\frac{GM}{R^3}}$$

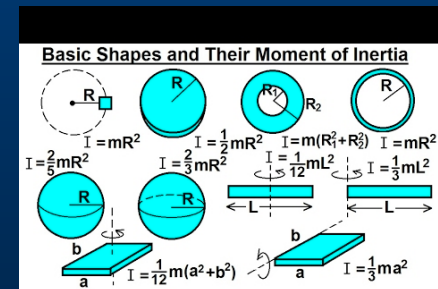
$$v_\phi = \sqrt{\frac{GM}{R}} = \omega_K R.$$

## Disk accretion

$$\frac{a_s}{v_\phi} \approx \frac{H}{R}.$$

Notice that since the particles move into Keplerian orbits there is no pressure gradient along  $R$ . The transport of angular momentum along the disk is associated with the moment of viscous forces:

$$\dot{M} \frac{d\omega_K}{dR} R^2 = 2\pi \frac{d}{dR} W_{r\phi} R^2.$$



# Disk accretion

Transport of angular momentum

$$\dot{M} \frac{d(\omega_K R^2)}{dR} = 2\pi \frac{d(W_{r\phi} R^2)}{dR}$$

Here,  $W_{r\phi}$  is the component of the viscous stress in the disk:

$$W_{r\phi} = -2\eta H R \frac{\partial \omega_K}{\partial R}.$$

The parameter  $\eta$  is the dynamic viscosity averaged over the  $z$ -coordinate. Notice that for a rigid body  $\partial \omega_K / \partial R = 0$  and the viscous stress vanishes.



# Viscosity

The viscosity of isotropic turbulence is (Landau & Lifshitz 2002):  $\eta = (1/3)\rho v_t l_t$ , where  $v_t$  and  $l_t$  are the characteristic velocity and scale of the turbulence, respectively. Shakura (1972) introduced the following expression to characterize the viscous fluid:

$$v_t l_t = \alpha a_s H,$$

where  $\alpha$  is the viscosity parameter. Since  $v_t < a_s$  and  $l_t < H$ , then  $\alpha \leq 1$ .

## Disk accretion

$$W_{r\phi} = -\frac{\dot{M}}{2\pi}\omega_K \left[ 1 - \left( \frac{R_d}{R} \right)^{1/2} \right] + W_{r\phi}(\text{in}),$$

where  $R_d$  is the radius of the inner edge of the disk and  $W_{r\phi}(\text{in})$  is the component of the tensor of viscous stress evaluated at  $R = R_d$ . As we have seen before, for a Schwarzschild black hole the last stable orbit is at  $R_d = 3R_{\text{Schw}}$ . For this last orbit, we can take:

$$W_{r\phi}(\text{in}) = 0.$$

The continuity equation can be written as:

$$\dot{M} = 2\pi\rho(2H)Rv_R,$$

with  $v_R$  the radial velocity of the matter in the accretion disk.

## Disk accretion

Far from the inner edge we have:

$$W_{r\phi} \approx -\frac{\dot{M}}{2\pi}\omega_K \approx 3\eta H\omega_K = \alpha P H.$$

From this equation

$$\frac{v_R}{v_\phi} \approx \alpha \left(\frac{H}{R}\right)^2.$$

## Disk accretion

The transport of angular momentum in the disk results in the generation of heat. We can express the heat produced per unit surface area of the disk per unit of time on each side as:

$$Q^+ = -\frac{1}{2}W_{r\phi}R\frac{d\omega}{dR} = \frac{3}{4}\omega W_{r\phi}. \quad \omega_K = \sqrt{\frac{GM}{R^3}}$$

This energy is carried away in the form of thermal radiation:

$$Q^- = \sigma_{\text{SB}}T^4,$$

In the steady state  $Q^+ = Q^-$ . If  $Q^+ > Q^-$ , the disk becomes thermally unstable.

## Disk accretion: basic equations

1.  $\omega = \omega_K = \left(\frac{GM}{R^3}\right)^{1/2}$  (Kepler's law).

2.  $\dot{M} = -2\pi\Sigma v_R R$  (continuity equation).

3.  $W_{r\phi} = -\frac{\dot{M}}{2\pi}\omega_K \left[1 - \left(\frac{R_d}{R}\right)^{1/2}\right] + W_{r\phi}(\text{in})$

4.  $P = \frac{\Sigma\omega^2 H}{6}$  (hydrostatic equilibrium).

5.  $W_{r\phi} = \alpha PH$  (viscous tensor).

6.  $Q^+ = -\frac{1}{2}W_{r\phi}R\frac{d\omega}{dR}$  (energy release).

$$\Sigma(R) = 2 \int_0^H \rho(R, z) dz$$

7.  $Q^- = \sigma_{\text{SB}}T^4$  (losses by radiation).

$$\Sigma \approx 2H\rho.$$

8.  $P = \frac{3}{2}\rho R_u(T_e + T_i) + \epsilon$  (equation of state, with  $\epsilon$  the energy density).

9.  $\sigma[\text{cm}^2] = \sigma_T + \sigma_{\text{ff}} \approx 6.65 \times 10^{-25} n + \frac{1.8 \times 10^{-25}}{T^{7/2}}$  (absorption cross section).

## Disk accretion: basic assumptions

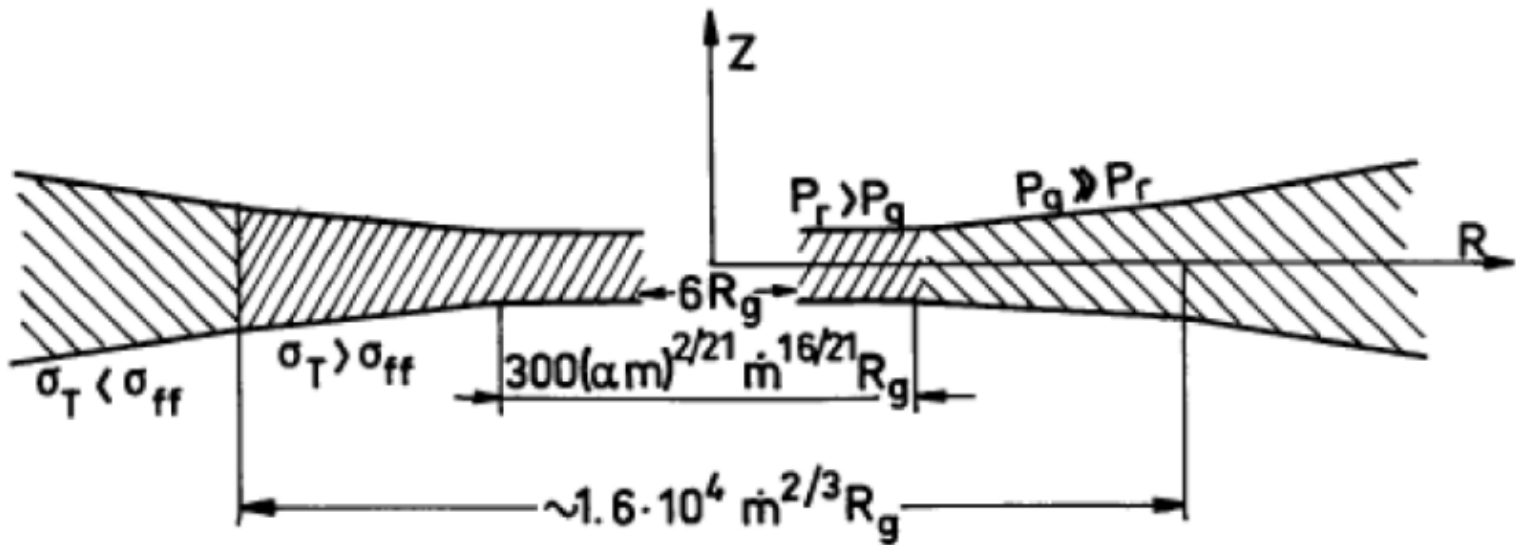
1. The gravitational field is determined by a black hole, and the self-gravity of the disk is ignored.
2. The disk lies in the equatorial plane of the hole.
3. The disk is steady.
4. The disk is axisymmetric.
5. The disk is geometrically thin in the sense that  $H/r \ll 1$ .
6. Rotational motion is dominant (Keplerian rotation);  $|v_r| \ll v_\phi$ .
7. Hydrostatic balance holds in the vertical direction.
8. The disk is optically thick in the vertical direction.
9. A specialized viscous law is adopted; *the  $r\phi$ -component of the viscous stress tensor is proportional to the pressure*. Other components are neglected.
10. Global magnetic fields are ignored.

## Disk accretion: regions

This is a system of 9 equations with 9 functions of  $R$  as solution. The solutions were found by Shakura & Sunyaev (1973). For fixed values of  $M$  and  $\dot{M}$ , the disk can be into three different regions:

- An outer region (large  $R$ ) where the gas pressure dominates over radiation pressure and opacity is controlled by free-free absorption.
- A middle region (smaller  $R$ ) where the gas pressure dominates over radiation pressure but the opacity is due to electron scattering.
- An inner region (very small  $R$ ) where radiation pressure dominates over gas pressure and the opacity is also due to electron scattering.

# Disk accretion: structure





## Disk accretion: luminosity

### Luminosity and spectrum of standard accretion disks

The energy carried with the radiation released in a ring of thickness  $dR$  of the accretion disk is:

$$dL(R) = 2Q^+ 2\pi R dR,$$

where the initial factor 2 is due to the two faces of the disk. This equation can be written as:

$$dL(R) = \frac{3}{2} \dot{M} \frac{d}{dR} \frac{GM}{R^2} \left( 1 - \sqrt{\frac{R_d}{R}} \right) dR.$$

## Disk accretion: luminosity

The power  $dL(R)$  corresponds to the work done by the gravitational field. Approximately half of this power is transformed into kinetic energy of the matter moving along  $\phi$  and the other half is transformed into heat:

$$dL_{\text{gr}} = \dot{M} \frac{d}{dr} \left( -\frac{GM}{2R} \right) dR = \frac{1}{2} \dot{M} \frac{GM}{R^2} dR.$$

## Disk accretion: luminosity

$$L_d = \int_{R_d}^{\infty} \frac{dL(R)}{dR} dR = \frac{\dot{M}GM}{2R}.$$

Adopting  $R_d = 3R_{\text{Schw}}$  and dividing by  $(dM/dt)c^2$  we get the efficiency of energy release in the disk accretion process:  $\sim 8\%$ . For a Kerr black hole, where  $R_d = R_g$ , the efficiency reaches  $\sim 42\%$ .

## Disk accretion: luminosity

$$Q^+ = \frac{3}{8\pi} \frac{\dot{M}GM}{R^3},$$

for  $R \gg R_d$ . Through the energy energy balance equation  $Q^+ = Q^- = \sigma_{\text{SB}}T^4$ , we can obtain the temperature distribution along the radial direction in the disk:

$$T(R) = \left( \frac{3}{8\pi\sigma_{\text{SB}}} \dot{M} \frac{GM}{R^3} \right)^{1/4} \propto R^{-3/4}.$$

## Disk accretion: luminosity

The total spectrum is the result of the superposition of the blackbody emission from each ring of temperature  $T(R)$ :

$$I_\nu = 2\pi \int_{R_d}^{R_{\text{out}}} B_\nu[T(R)] R dR,$$

with

$$B_\nu(T) = \frac{2\pi h}{c^2} \left( \frac{kT}{h} \right)^3 \frac{x^3}{e^x - 1}; \quad x \equiv \frac{h\nu}{kT},$$

## Disk accretion: luminosity

$$R_{\text{out}} \gg R_{\text{d}}$$

$$I_{\nu} = \frac{16\pi^2 R_{\text{d}}^2}{c^2} \left( \frac{kT_{\text{d}}}{h} \right)^{8/3} h\nu^{1/3}.$$

The typical temperature can be obtained from:

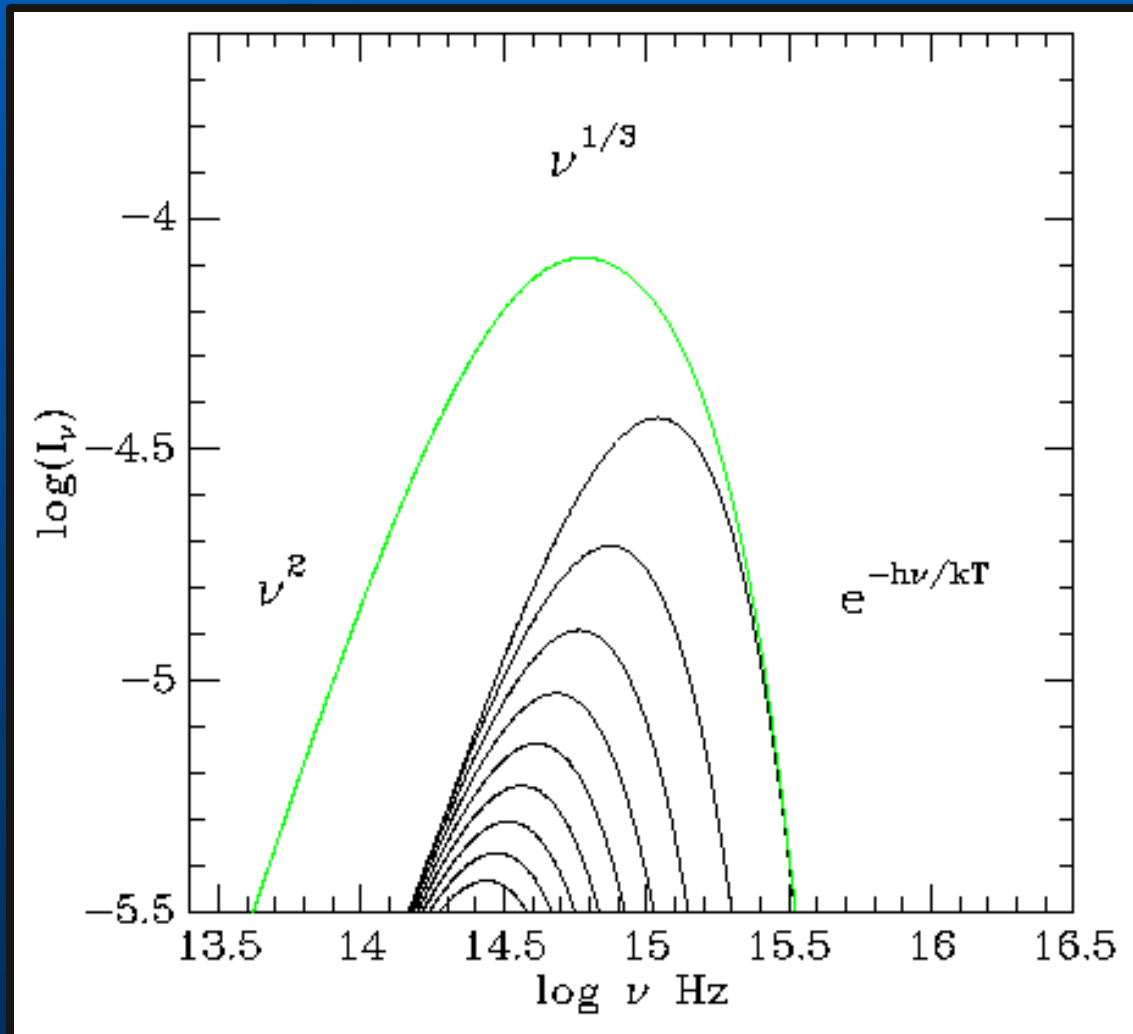
$$\frac{1}{2} \dot{M} \frac{GM}{R_{\text{d}}} = 2\pi R_{\text{d}}^2 \sigma_{\text{SB}} T^4$$

The result is:

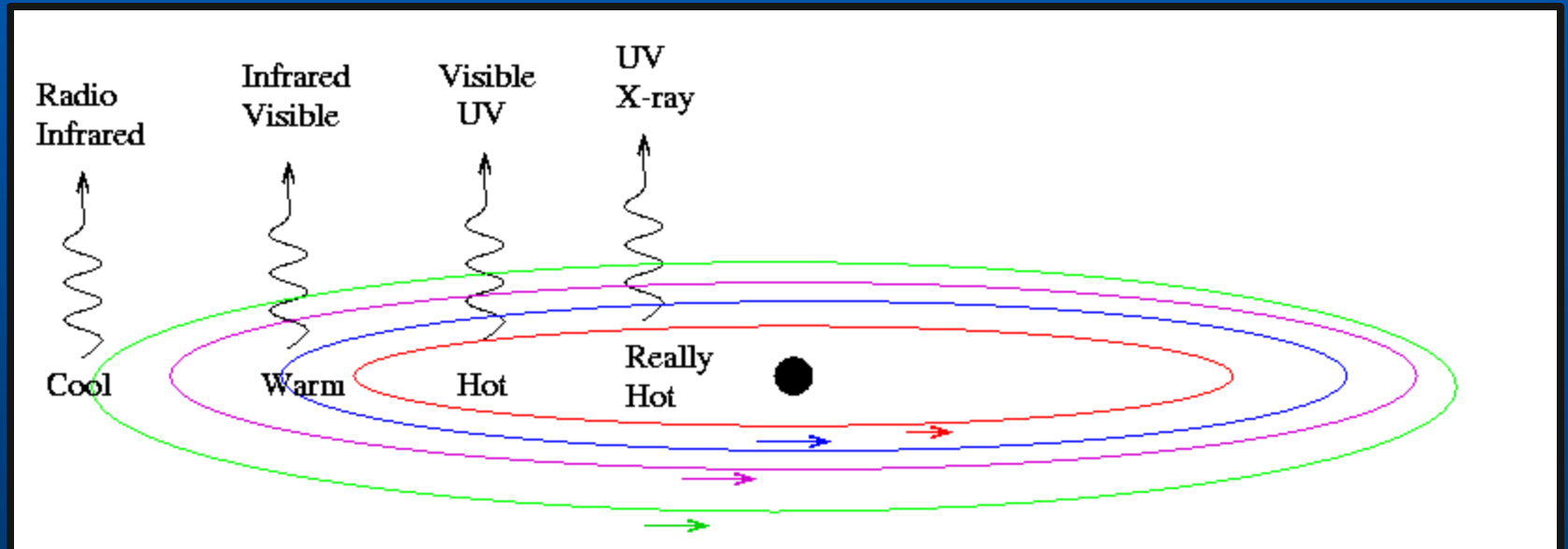
$$T = \left( \frac{\dot{M}GM}{4\pi R_{\text{d}}^3 \sigma_{\text{SB}}} \right)^{1/4}.$$

This yields temperatures of  $\sim 10^7$  K for stellar mass black holes in binary systems ( $\dot{M} \sim 10^{18}$  g s<sup>-1</sup>).

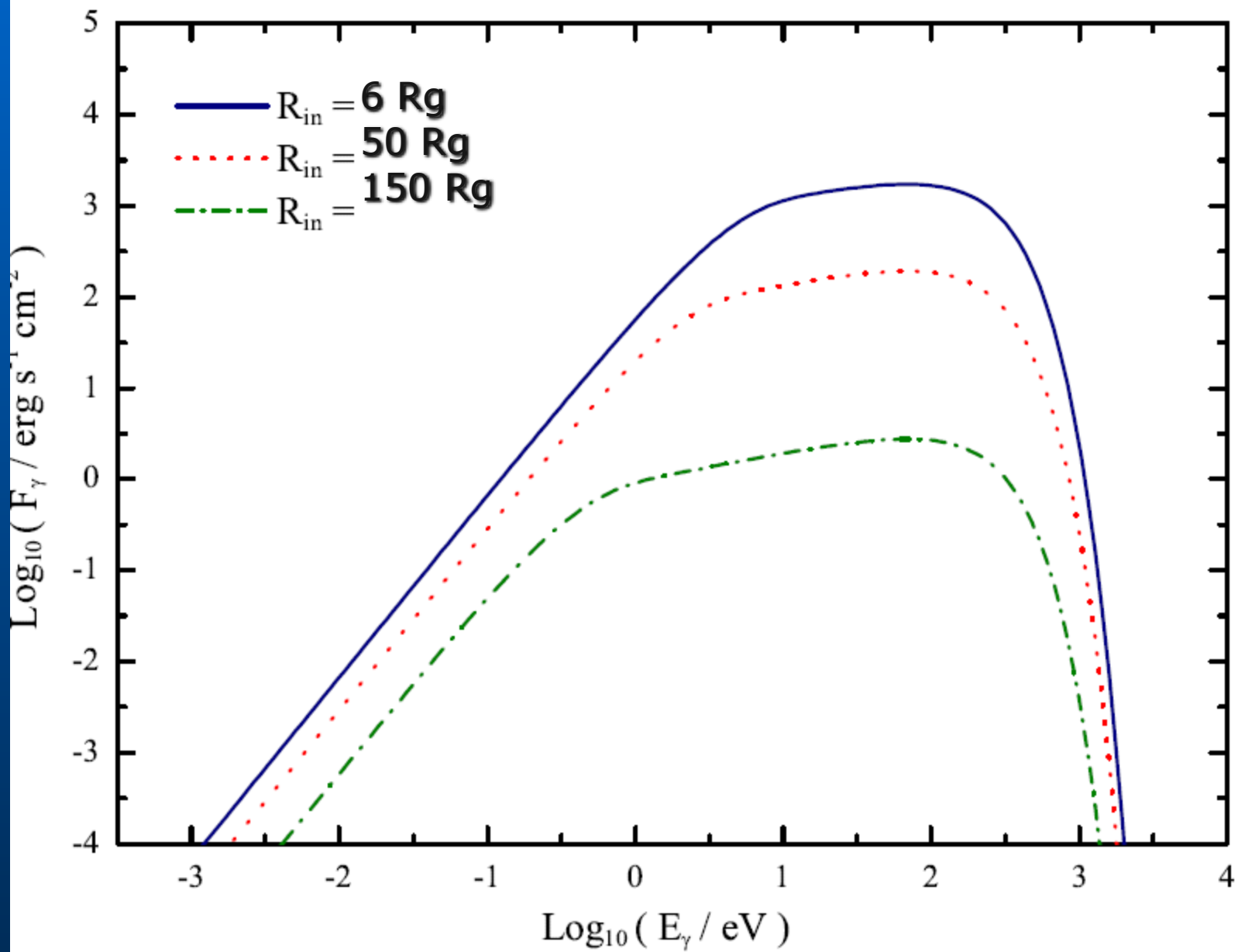
# Disk accretion: luminosity

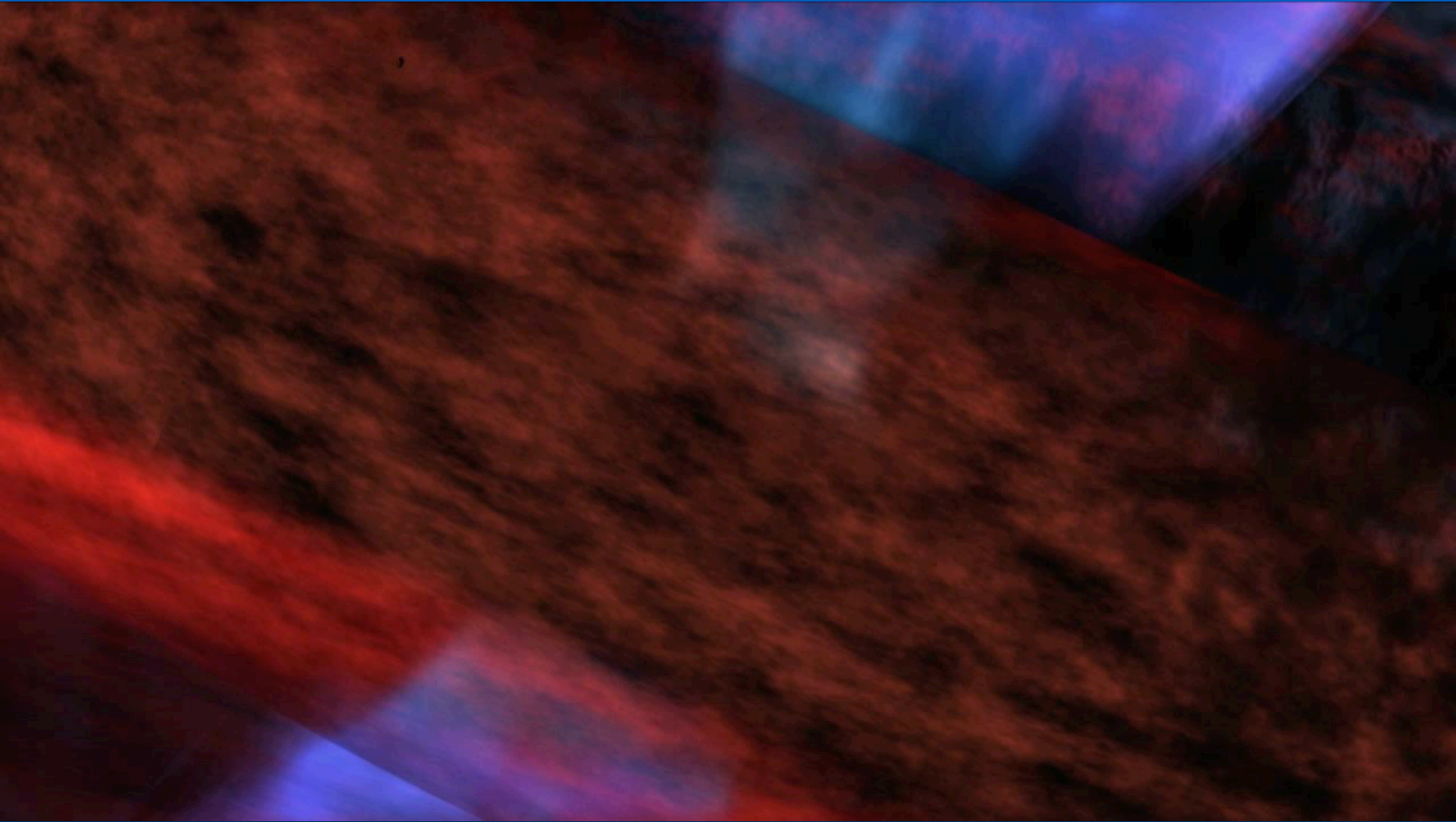


# Disk accretion: luminosity

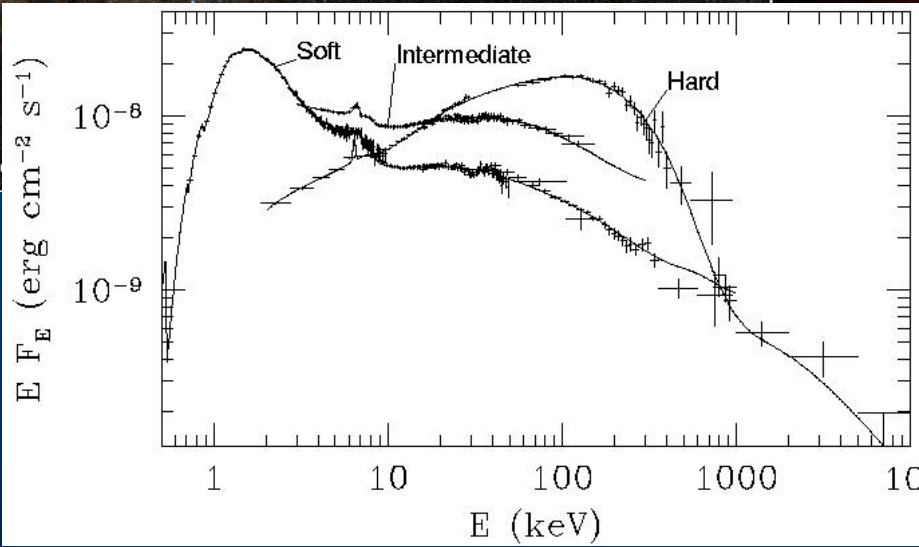
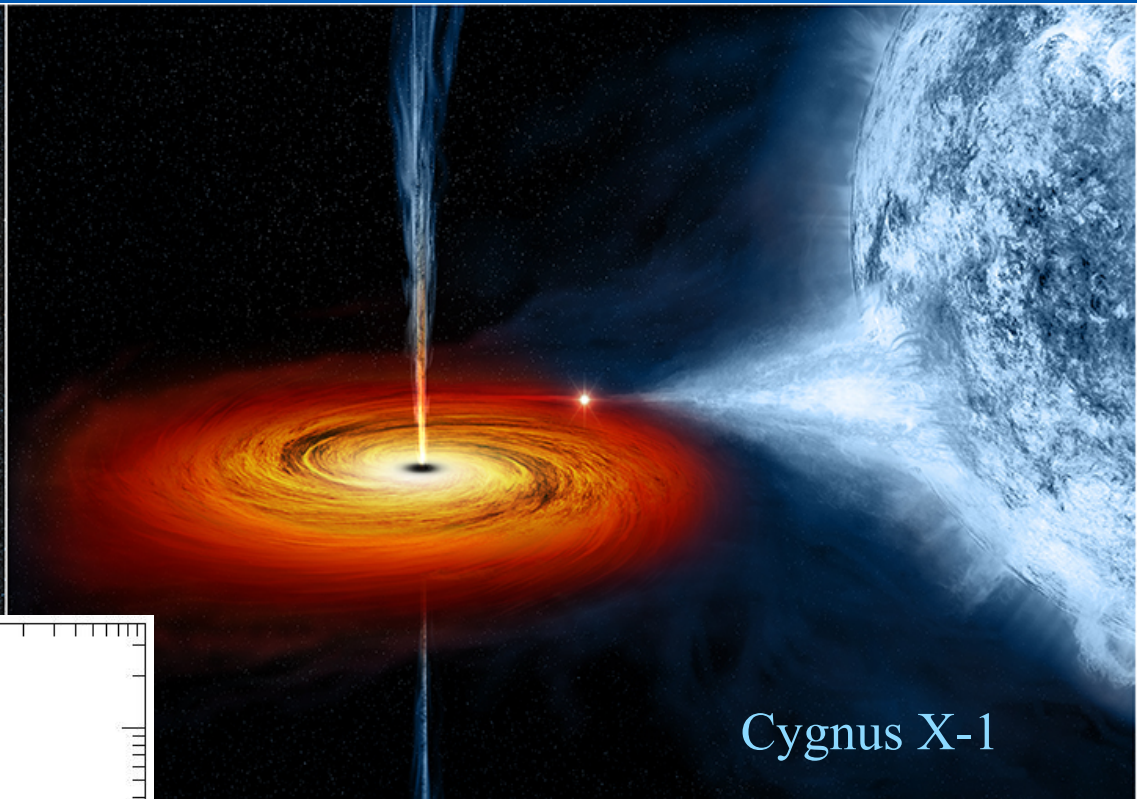
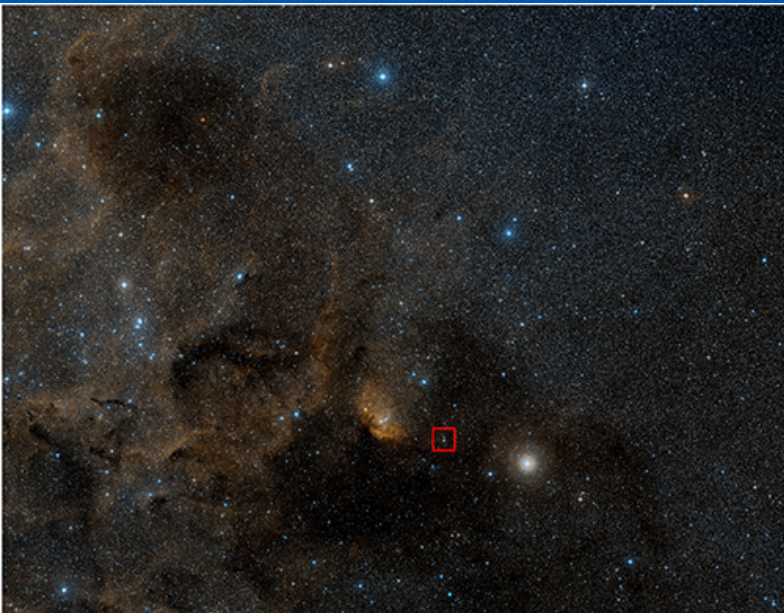








# The need for new solutions



Emission above what is expected from a thin disk with  $T \sim 10^7$  K ( $\sim$  a few keV).



## Disk accretion: hot accretion flows and ADAF

Shapiro, Lightman, and Eardly (1976) found a self-consistent solution for the hydrodynamic equations of an accreting flow onto a compact object, including both rotation and viscosity. This solution has the characteristic that the plasma has two-temperatures.

The ion temperature ( $T \sim 10^{12}$  K) is much higher than the electron temperature ( $T \sim 10^9$  K). The plasma is optically thin and the radiation has a power-law spectrum in X-rays, consistent with what is observed in sources like Cygnus X-1. However, the solution is thermally unstable.

## Disk accretion: ADAF

Thermally stable solutions were found by Begelman and Meier (1982) in a super-Eddington accretion regime (the disk results optically thick) and by Ichimaru (1977) and Narayan and Yi (1994a,b, 1995a,b). The latter solution corresponds to sub-Eddington accretion of a low-density gas.

The energy released by viscosity is stored in the plasma, which is advected and swallowed by the black hole. The plasma is optically thin and with two temperatures. This type of solution describes what is known as advection-dominated accretion flows (ADAFs).

## Disk accretion: ADAF

In general, we can summarize the characteristics of an ADAF as follows:

1. the radial velocity is a considerable fraction of the free-fall velocity so accretion is fast,
2. the rotation velocity is sub-Keplerian,
3. the gas is expected to be hot since it has no time to cool before being accreted,
4. the typical height scale is  $H \sim a_s/\Omega_K \sim r$ —the flow is then quasi-spherical.

## Disk accretion: ADAF

The different accretion regimes in an ADAF are determined by the parameter  $f$  defined as:

$$f = \frac{Q^+ - Q^-}{Q^+} \equiv \frac{Q_{\text{adv}}}{Q^+},$$

i.e. as the ratio between the advected energy and the energy released through viscosity. Different values of  $f$  correspond to different types of accretion.

## Disk accretion: ADAF

- $f \ll 1$ : in this case  $Q^+ \approx Q^- \gg Q_{\text{adv}}$  and all the energy released by viscosity is radiated. This regime corresponds to thin disks and two-temperature solutions such as that of Shapiro et al. (1976).
- $f \approx 1$ : here  $Q_{\text{adv}} \approx Q^+ \gg Q^-$ , the cooling is negligible and the flow is ADAF-like.
- $|f| \gg 1$ : corresponds to  $-Q_{\text{adv}} \approx Q^- \gg Q^+$ . The situation is like in the Bondi-Hoyle regime.



## Disk accretion: ADAF

The equations that describe an ADAF are the usual hydrodynamic equations for a viscous accretion flow. It is convenient now to work in spherical coordinates  $(r, \theta, \phi)$ . We shall assume that the system has azimuthal symmetry, so that  $\partial/\partial\phi = 0$ , and has reached the steady state.

$\partial/\partial\phi = 0$ , and has reached the steady state.

using  $(r, \theta, \phi)$ : use usual hydrodynamic equations, use spherical coordinates; so, use

If we consider spherical coordinates  $(R, \theta, \phi)$  the three components of the Euler equation and the energy conservation can be written as:

$$\rho \left( v_R \frac{\partial v_R}{\partial R} - \frac{v_\phi^2}{R} \right) = -\frac{GM\rho}{R^2} - \frac{\partial p}{\partial R} + \frac{\partial}{\partial R} \left[ 2\nu\rho \frac{\partial v_R}{\partial R} - \frac{2}{3}\nu\rho \left( \frac{2v_R}{R} + \frac{\partial v_R}{\partial R} \right) \right] \\ + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \frac{\nu\rho}{R} \frac{\partial v_R}{\partial \theta} \right) + \frac{\nu\rho}{R} \left[ 4R \frac{\partial}{\partial R} \left( \frac{v_R}{R} \right) + \frac{\cot \theta}{R} \frac{\partial v_R}{\partial \theta} \right],$$

$$\rho \left( -\frac{\cot \theta}{R} v_\phi^2 \right) = -\frac{1}{R} \frac{\partial p}{\partial \theta} + \frac{\partial}{\partial R} \left( \frac{\nu\rho}{R} \frac{\partial v_R}{\partial \theta} \right) \\ + \frac{1}{R} \frac{\partial}{\partial \theta} \left[ \frac{2\nu\rho}{R} v_R - \frac{2\nu\rho}{3} \left( \frac{2v_R}{R} + \frac{\partial v_R}{\partial R} \right) \right] + \frac{3\nu\rho}{R^2} \frac{\partial v_R}{\partial \theta},$$

$$\rho \left( v_R \frac{\partial v_\phi}{\partial R} + \frac{v_\phi v_R}{R} \right) = \frac{\partial}{\partial R} \left[ \nu\rho R \frac{\partial}{\partial R} \left( \frac{v_\phi}{R} \right) \right] + \frac{1}{R} \frac{\partial}{\partial \theta} \left[ \frac{\nu\rho \sin \theta}{R} \frac{\partial}{\partial \theta} \left( \frac{v_\phi}{\sin \theta} \right) \right] \\ + \frac{\nu\rho}{R} \left[ 3R \frac{\partial}{\partial R} \left( \frac{v_\phi}{R} \right) + \frac{2 \cot \theta \sin \theta}{R} \frac{\partial}{\partial \theta} \left( \frac{v_\phi}{\sin \theta} \right) \right],$$

$$\rho \left( v_R \frac{\partial \varepsilon}{\partial R} - \frac{p}{\rho^2} v_R \frac{\partial \rho}{\partial R} \right) = -\frac{2f\nu\rho}{3} \left[ \frac{1}{R^2} \frac{\partial}{\partial R} (R^2 v_R) \right]^2 + 2f\nu\rho + \left\{ \left( \frac{\partial v_R}{\partial R} \right)^2 + 2 \left( \frac{v_R}{R} \right)^2 \right. \\ \left. + \frac{1}{2} \left( \frac{1}{R} \frac{\partial v_R}{\partial \theta} \right)^2 + \frac{1}{2} \left[ R \frac{\partial}{\partial R} \left( \frac{v_R}{R} \right) \right]^2 + \frac{1}{2} \left[ \frac{\sin \theta}{R} \frac{\partial}{\partial \theta} \left( \frac{v_\phi}{\sin \theta} \right) \right]^2 \right\}.$$

## Disk accretion: ADAF

Narayan and Yi (1995a) found the following self-similar solutions for these equations:

$$v_R = R\Omega_K(R)v(\theta), \quad \Omega_K(R) = \left(\frac{GM}{R^3}\right)^{1/2}.$$

$$v_\theta = 0,$$

$$v_\phi = R\Omega_K(R)\Omega(\theta),$$

$$c_s = R\Omega_K(R)c_s(\theta),$$

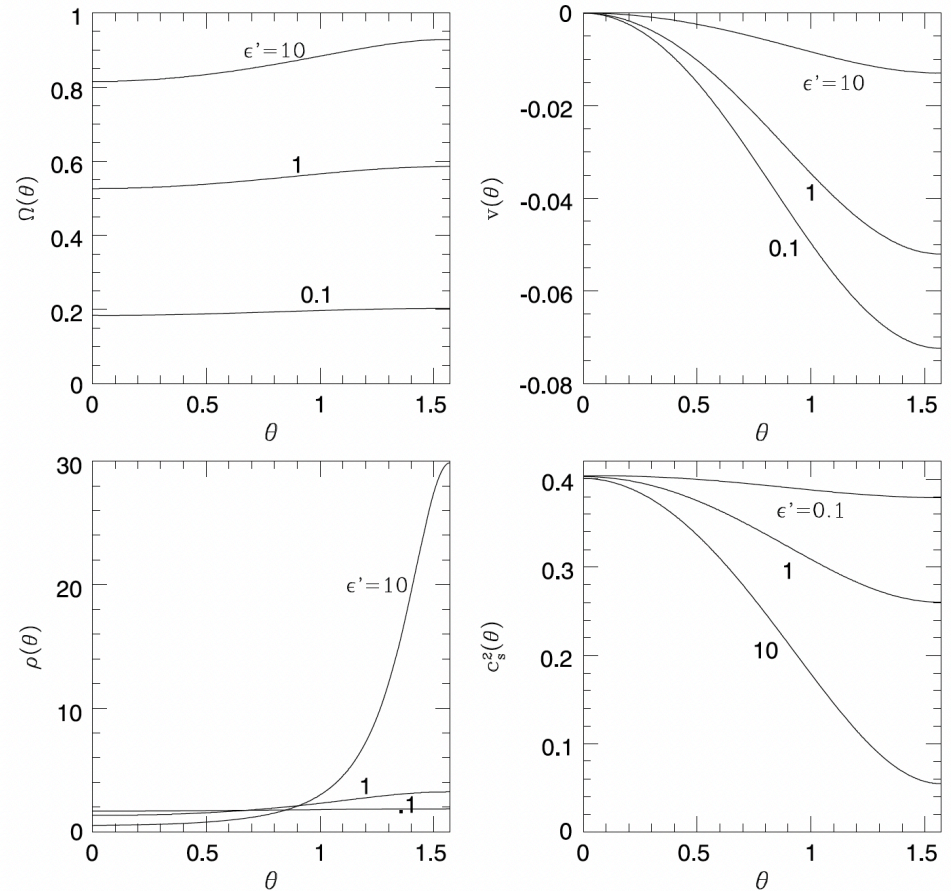
$$\rho = R^{-3/2}\rho(\theta).$$

$$v = \frac{\alpha a_s^2}{\Omega_K}$$

We also need an equation of state

# Disk accretion: ADAF

$$\varepsilon' \equiv \frac{\varepsilon}{f} = \frac{1}{f} \left( \frac{5/3 - \gamma}{\gamma - 1} \right),$$



Angular velocity  $\Omega$ , radial velocity  $v$ , mass density  $\rho$ , and speed of sound  $c_s$  at fixed radius as a function of the polar angle  $\theta$ , for  $\alpha = 0.1$  and several values of the parameter  $\varepsilon$ . From Narayan et al. (1998b).

## Disk accretion: 2T ADAF

- The total pressure has contributions from both the gas and the magnetic field:

$$p = p_g + p_m.$$

The magnetic pressure is:

$$p_m = \frac{B^2}{8\pi},$$

and the gas pressure, in the case of an ideal gas of density  $n$  and temperature  $T$ ,

$$p_g = nkT.$$

The first hypothesis is that the magnetic pressure is a fixed fraction of the gas pressure:

$$p_m = (1 - \beta)p, \quad p_g = \beta p.$$

The value  $\beta = 0.5$  corresponds to strict equipartition.

## Disk accretion: 2-T ADAF

- A second hypothesis is that the temperature of ions and temperature of electrons are different. Then, the gas pressure becomes:

$$p_g = \beta \rho c_s^2 = \frac{\rho}{\mu_i m_H} k T_i + \frac{\rho}{\mu_e m_H} k T_e,$$

where  $m_H$  is the hydrogen mass and  $\mu_{i,e}$  the molecular weight of ions and electrons, respectively.

## Disk accretion: 2-T ADAF

- There is a preferential heating of ions. Because of the large difference in mass, it is assumed that the energy released by viscosity is transferred to ions and just a small fraction  $\delta \ll 1$  goes to electrons. Usually, it is assumed  $\delta \sim 10^{-3} \sim m_e/m_p$ . In such a case, the result will be  $T_i \gg T_e$ , where typically  $T_i \sim 10^{12}$  K and  $T_e \sim 10^9$  K. Even if both types of particles receive the same amount of energy, electrons will cool more efficiently, leading to  $T_i \gg T_e$  in any case.
- ADAF models assume that there is no thermal coupling between ions and electrons, and the only relevant interaction is Coulombian.

## Disk accretion: 2-T ADAF

- The resulting spectrum of the ADAF will result from the operation of the different cooling mechanisms. For electrons the most relevant mechanisms are synchrotron radiation, Bremsstrahlung, and inverse Compton scattering:

$$Q_e^- = Q_{\text{Br}}^- + Q_{\text{synchr}}^- + Q_{\text{IC}}^-.$$

Photons produced by Bremsstrahlung and synchrotron process can be up-scattered by electrons, in addition to those coming from external fields. Then,  $Q_{\text{IC}}^-$  can be written as:

$$Q_{\text{IC}}^- = Q_{\text{IC,Br}}^- + Q_{\text{IC,synchr}}^- + Q_{\text{IC,ext}}^-.$$

In the steady state the energy gained by the ions through the viscous heating must be equal to the energy transferred to the electrons plus the advected energy:

$$Q^+ = Q_{\text{adv}} + Q_{ie} = fQ^+ + Q_{ie}.$$

This assumes that the ions have no radiative losses.



## Disk accretion: 2-T ADAF

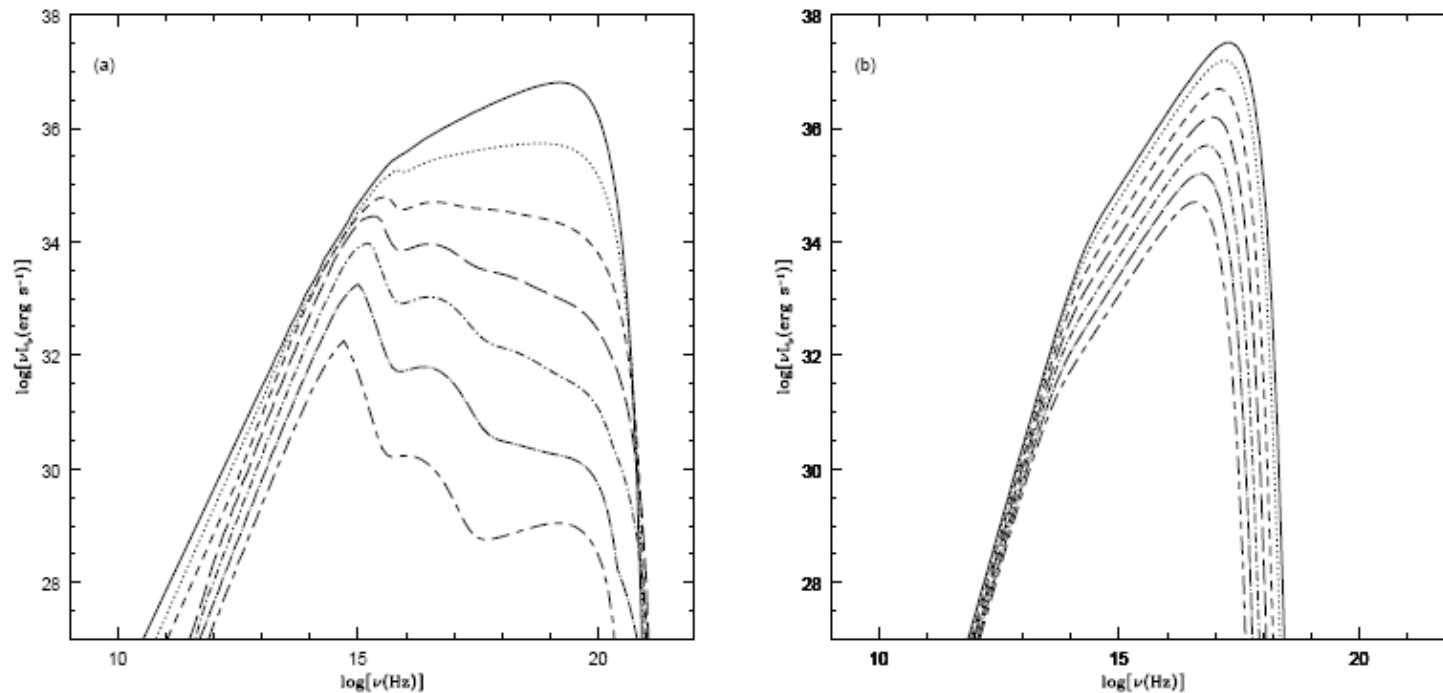
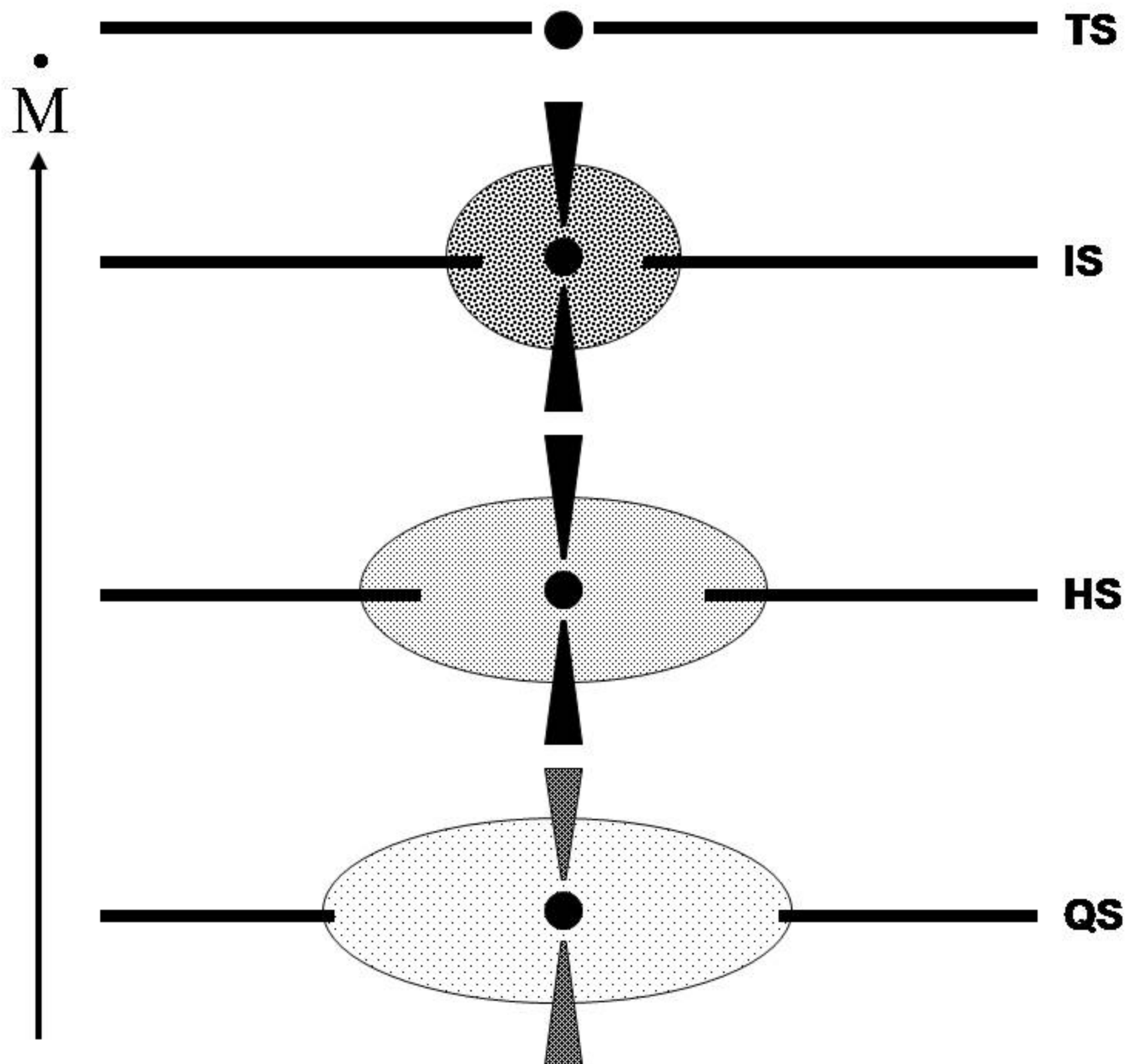
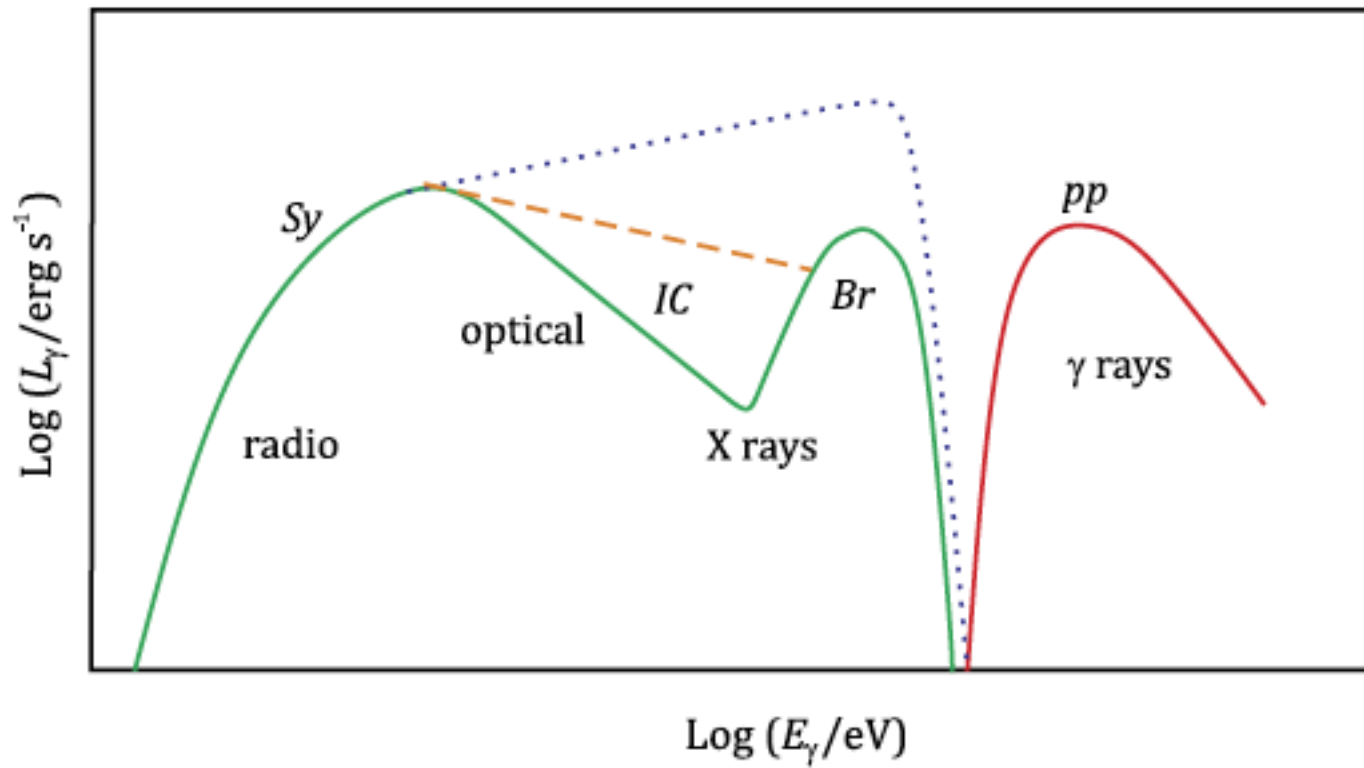


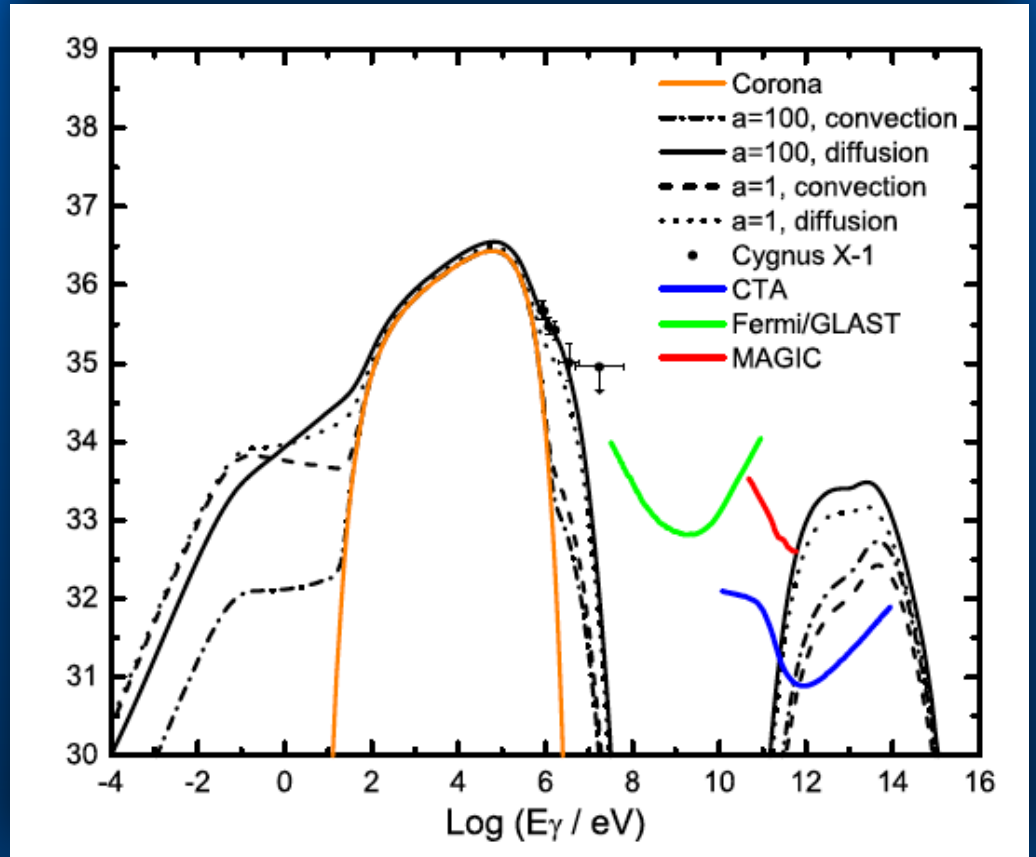
Figure 32. Left: ADAF spectra from a  $10 M_\odot$  black hole and different accretion rates. *Right*: Thin disk spectra for the same accretion rates and black hole.



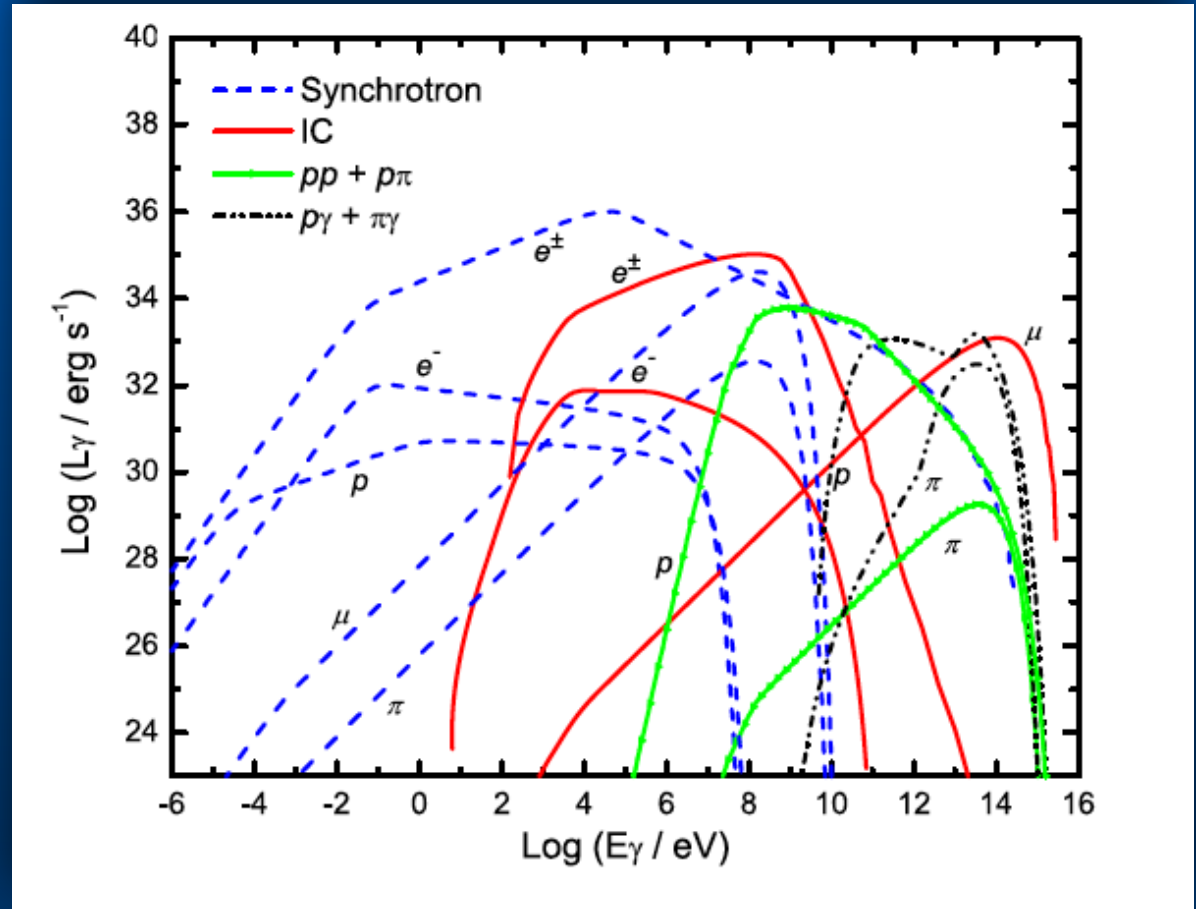
# Generic SED of an ADAF



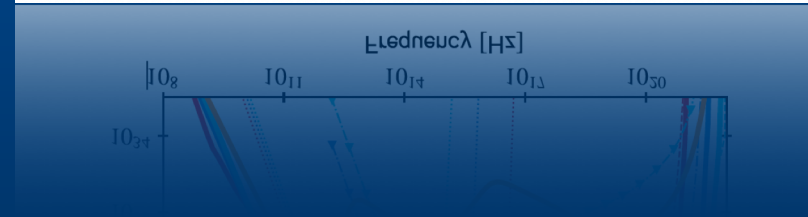
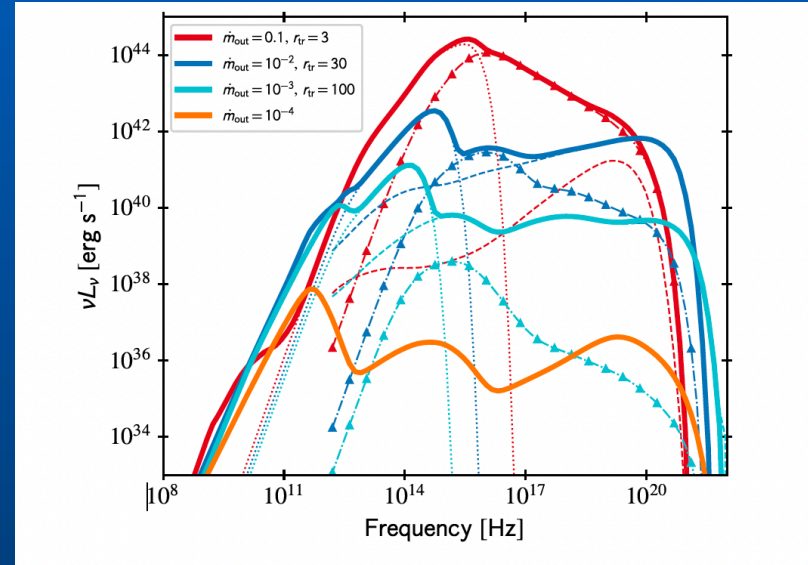
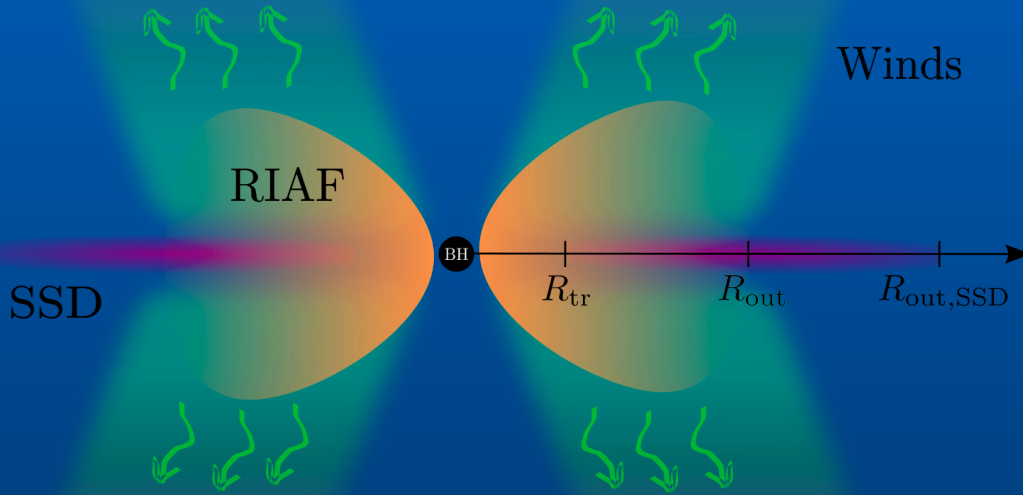
Non-thermal contributions to the radiative spectrum in a corona model for Cygnus X-1. Figure from Romero et al. (2010).



Non-thermal contributions to the radiative spectrum in a corona model with a relativistic proton-to-electron power ratio of 100. From Romero et al. (2010).

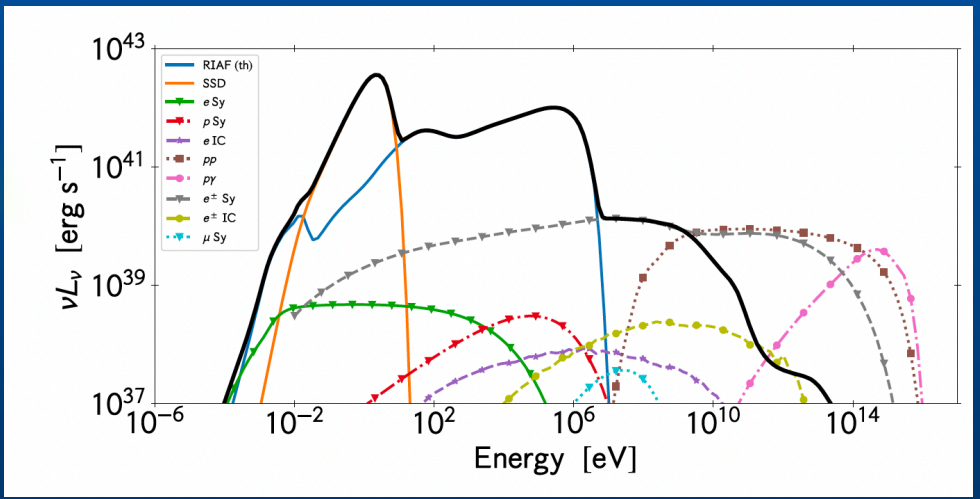
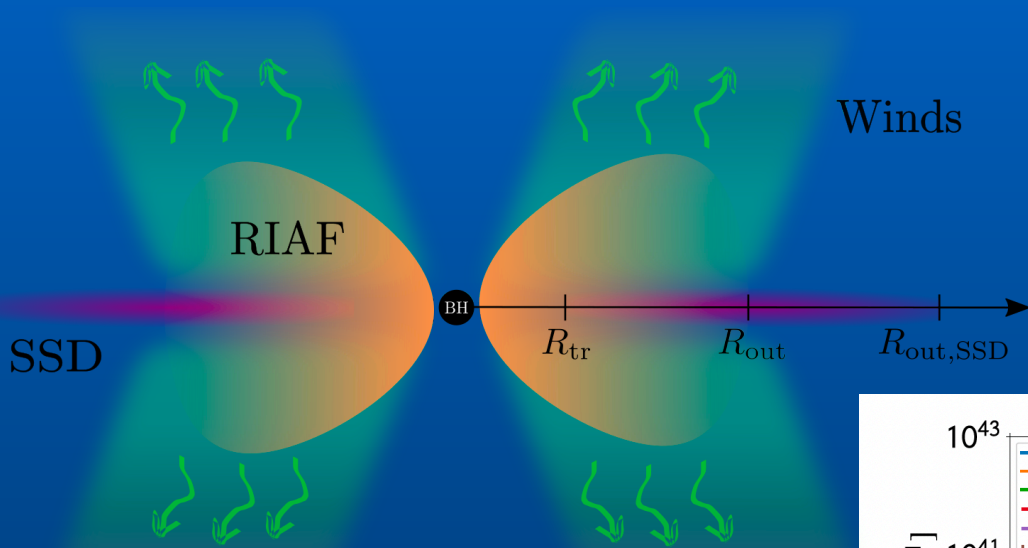


# Radiatively inefficient accretion flows (RIAFs)



Thermal emission from the accretion flow around a supermassive black hole of mass  $M_{BH} = 10^8 M_{sol}$  for four different models (from Gutierrez, Vieyro & Romero 2021)

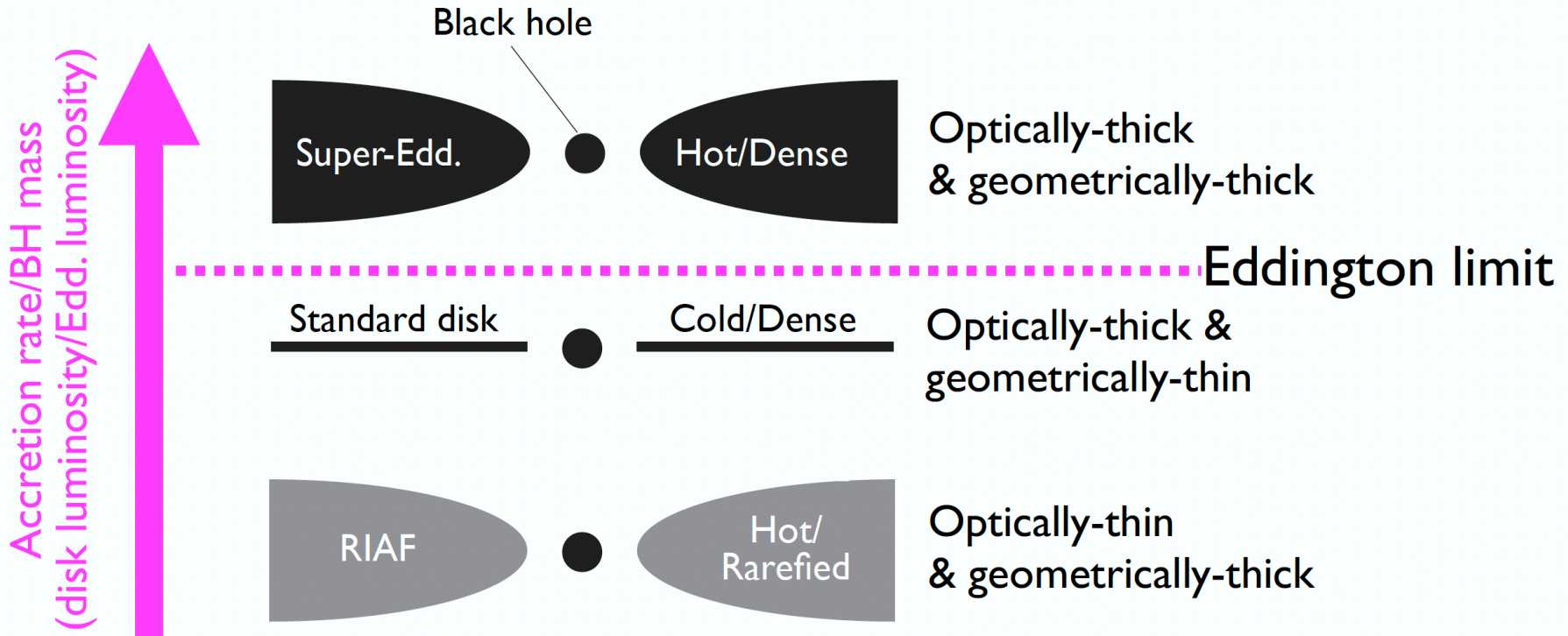
# Radiatively inefficient accretion flows (RIAFs)



Spectral energy distribution of a supermassive RIAF with non-thermal particles. Accretion rate:  $\dot{m}_{\text{out}} = 10^{-2}$  (from Gutierrez, Vieyro & Romero 2021)

# Three Accretion Modes

Shakura & Sunyaev 73; Ichimaru 77; Abramowicz+ 88; Narayan & Yi 94;

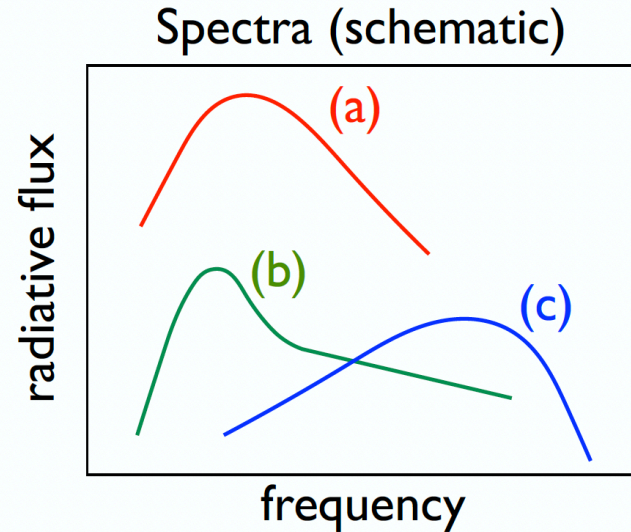


Accretion disks are divided into three modes by depending on the accretion rate (disk luminosity)

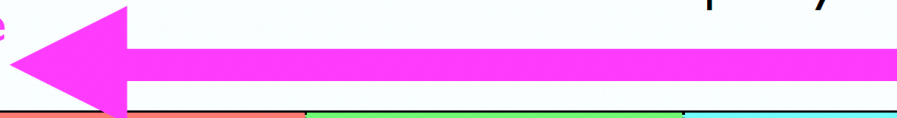


# Three Accretion Modes

Different spectral states imply the existence of different accretion modes

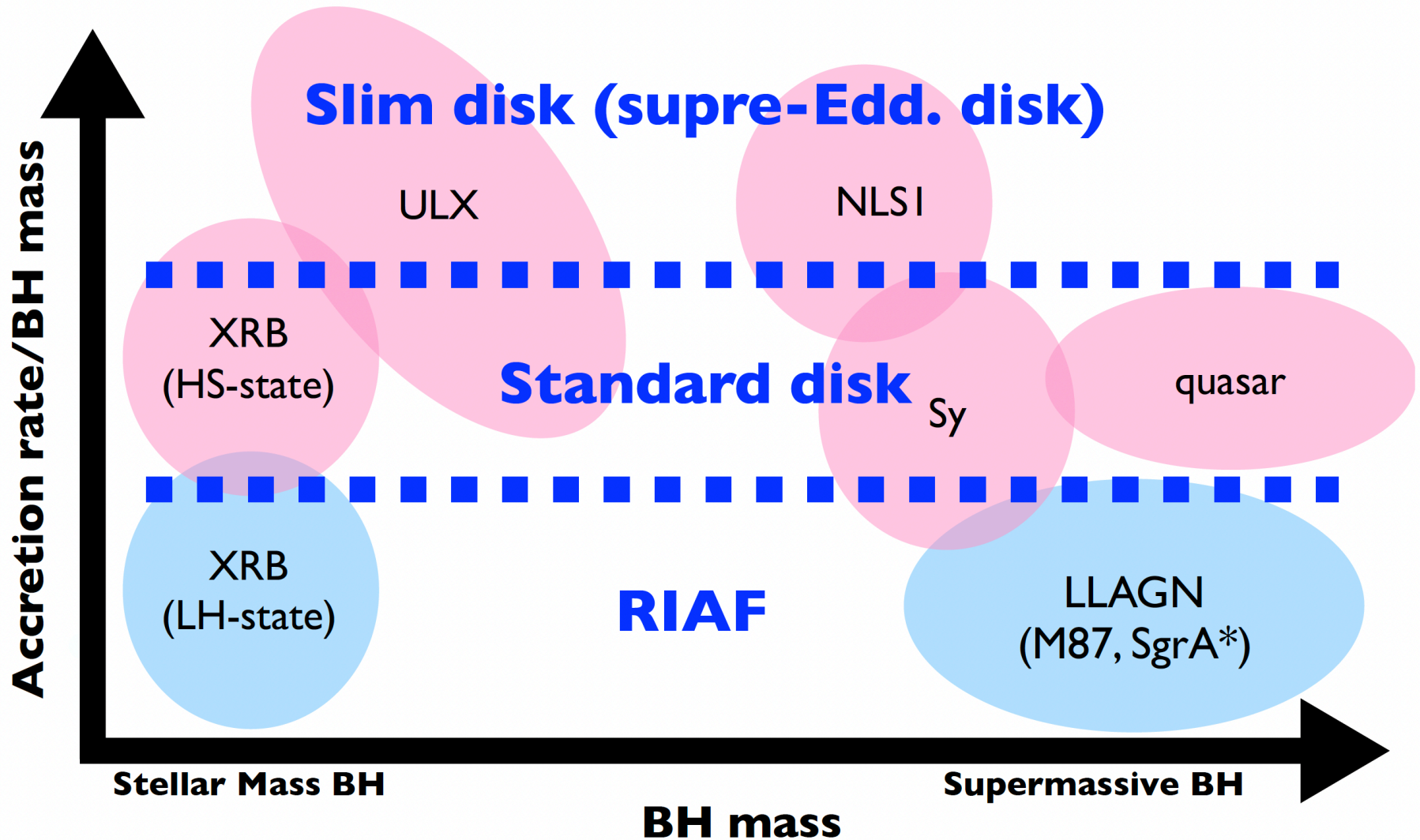


Mass accretion rate  
(disk luminosity)

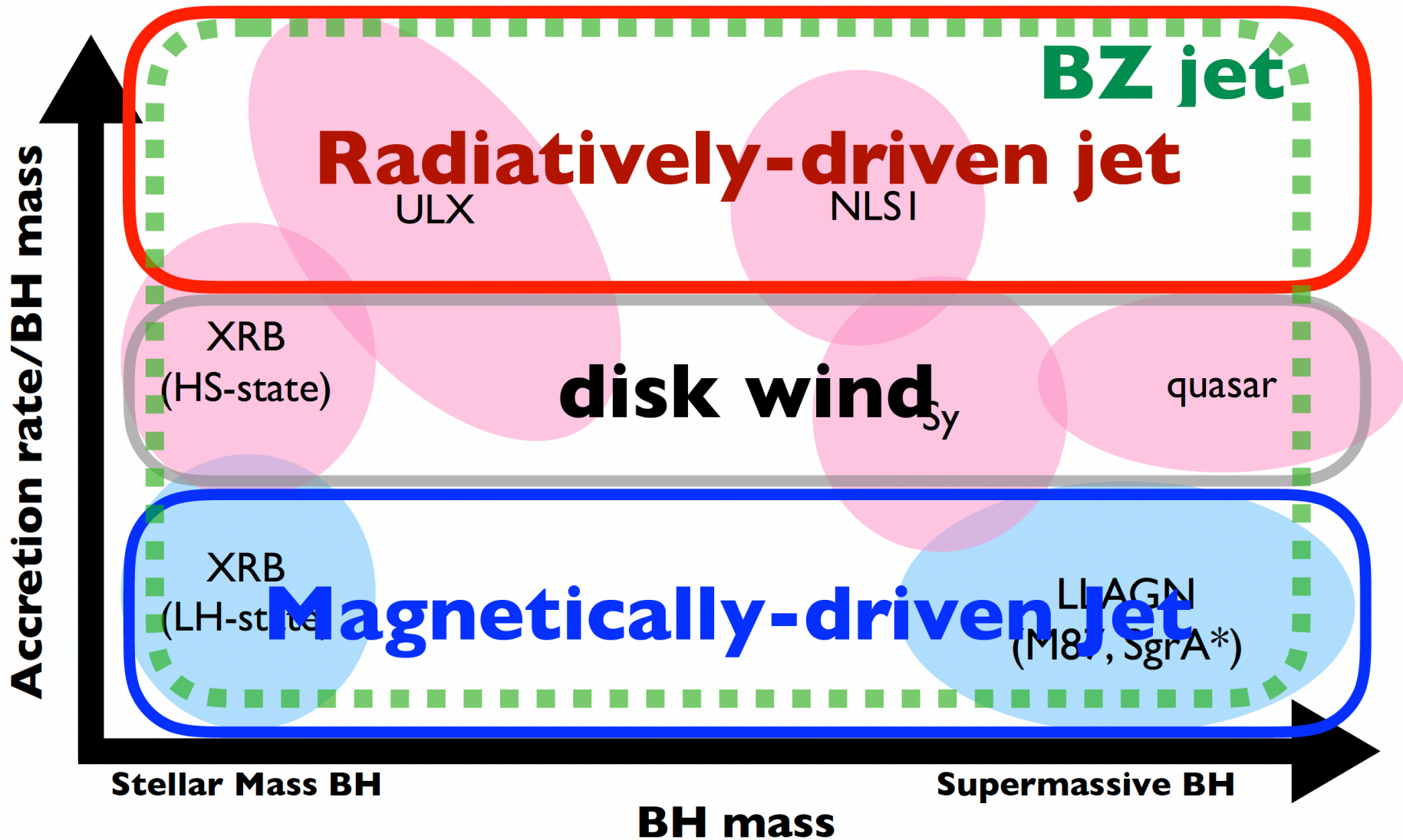


	(a) Slim disk	(b) Standard disk	(c) RIAF/ADAF
Temperature (10Msun)	$\sim 10^8\text{K}$	$\sim 10^{6-7}\text{K}$	$10^9\text{K}$
Optical thickness	thick	thick	thin

# BH mass vs Accretion rate

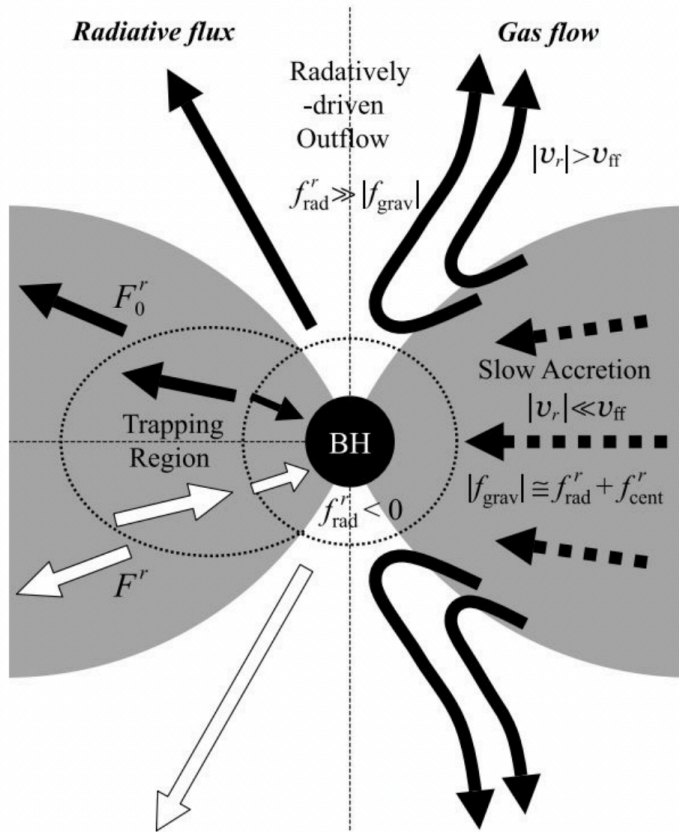


# BH mass vs Accretion rate





# Photon trapping in super critical accretion



diffusion timescale  $t_{\text{diff}} = \frac{H}{c/3\tau}$

accretion timescale  $t_{\text{acc}} = -\frac{r}{v_r}$

Condition for photon-trapping in the disk:

$$H/(c/3\tau) \gtrsim -r/v_r$$

$$r_{\text{trap}} = 3 \frac{\dot{M}}{\dot{M}_{\text{crit}}} \frac{H}{R} r_g$$

## Super accreting disks

The critical radius:

$$\text{Vertical Force} = -\frac{GMz}{R^3} + \frac{\sigma_T}{m_p c} F,$$

$$F = \sigma T^4 = 3GM\dot{M}/(8\pi r^3).$$

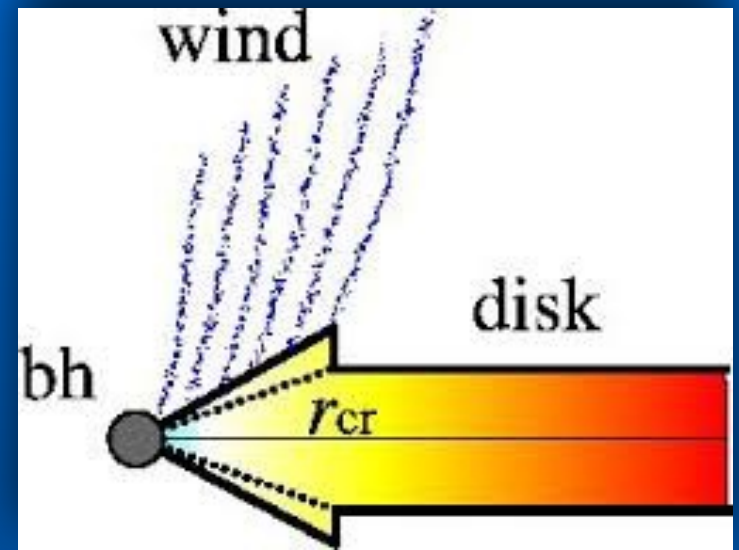
$$r_{\text{cr}} = \frac{9\sqrt{3}\sigma_T}{16\pi m_p c} \dot{M}_{\text{input}},$$

$$r_{\text{cr}} = 5.71 \times 10^5 \frac{M}{M_{\odot}} \frac{\dot{M}_{\text{input}}}{\dot{M}_{\text{crit}}} \text{ cm}$$

or

$$r_{\text{cr}} = \frac{9\sqrt{3}}{8} \dot{m} r_g \sim 1.95 \dot{m} r_g,$$

where  $\dot{m} \equiv \dot{M}_{\text{input}}/\dot{M}_{\text{crit}}$  and  $r_g = 2GM/c^2$ .



Fukue 2004

$$\dot{M}_{\text{crit}} \equiv \frac{L_E}{c^2} = 1.39 \times 10^{17} \frac{M}{M_{\odot}} \text{ g s}^{-1}$$

# Super accreting disks

$$H = \begin{cases} \frac{3\kappa f_{\text{in}}}{32\pi c} \dot{M}_{\text{input}} & \text{for } r \geq r_{\text{cr}} \\ \sqrt{c_3} r & \text{for } r \leq r_{\text{cr}}, \end{cases}$$

$$f_{\text{in}} = 1 - \sqrt{r_{\text{in}}/r}$$

where  $c_3$  is a numerical factor of the order of unity

$$\sqrt{c_3} = H/r = \tan \delta.$$

$$\sigma T_{\text{eff}}^4 = \begin{cases} \frac{3GM\dot{M}_{\text{input}}}{8\pi r^3} f_{\text{in}} & \text{for } r \geq r_{\text{cr}} \\ \frac{3}{4} \sqrt{c_3} \frac{L_E}{4\pi r^2} & \text{for } r \leq r_{\text{cr}}. \end{cases}$$

$$L_{\text{disk}} = \int_{r_{\text{in}}}^{r_{\text{cr}}} 2\sigma T_{\text{eff}}^4 2\pi r dr + \int_{r_{\text{cr}}}^{\infty} 2\sigma T_{\text{eff}}^4 2\pi r dr$$

$$= \left\{ \begin{array}{l} \frac{2}{3\sqrt{3}} \\ \frac{3}{4}\sqrt{c_3} \end{array} \right\} L_E \ln \frac{r_{\text{cr}}}{r_{\text{in}}} + \frac{2}{3\sqrt{3}} L_E,$$

$$L_{\nu} = 2 \int_{r_{\text{in}}}^{r_{\text{out}}} \pi B_{\nu}(r) 2\pi r dr,$$

where a factor 2 means both sides of the disk, and

$$B_{\nu}(r) = \frac{2h}{c^2} \frac{\nu^3}{e^{h\nu/k_B T_{\text{eff}}(r)} - 1}.$$

Mineshige and Ohsuga

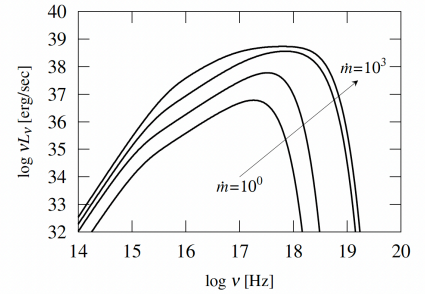
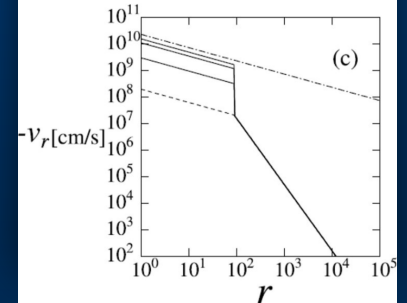
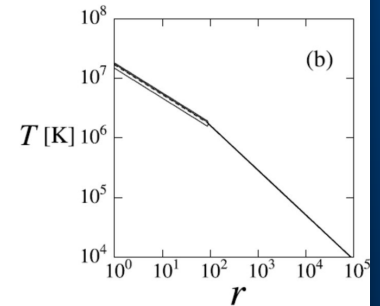
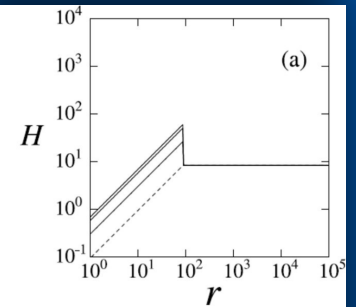


Figure 3. The emergent spectra of the calculated disks. The parameters are  $M = 10M_{\odot}$  and  $\dot{M}/(L_E/c^2) = 1, 10, \dots, 10^3$ . (After Watarai et al. 2000.)



$$\dot{M} = \begin{cases} \dot{M}_{\text{input}} & \text{for } r \geq r_{\text{cr}} \\ \frac{16\pi c m_{\text{p}}}{9\sqrt{3}\sigma_{\text{T}}} r = \dot{M}_{\text{input}} \frac{r}{r_{\text{cr}}} & \text{for } r \leq r_{\text{cr}}. \end{cases}$$

$$\dot{M}(r) = \frac{16\pi c m_{\text{p}}}{9\sqrt{3}\sigma_{\text{T}}} r$$

inside  $r_{\text{cr}}$ . Thus, the wind mass-loss rate via radiatively-driven winds must be

$$\dot{M}_{\text{wind}} = \dot{M}_{\text{input}} - \dot{M}(r),$$

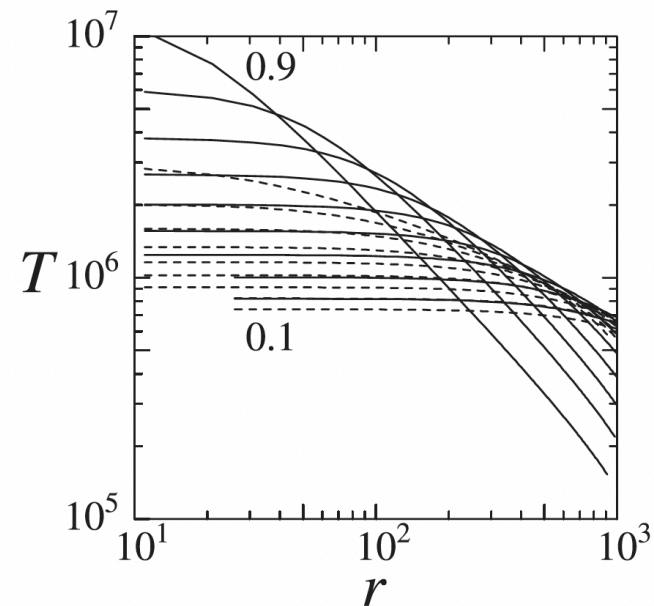
where  $\dot{M}_{\text{input}}$  is the accretion rate at the outer edge of the disk (and at the critical radius).

The total wind mass-loss rate  $\dot{M}_{\text{wind}}$  is evaluated as

$$\dot{M}_{\text{wind}} = \dot{M}_{\text{input}} \left( 1 - \frac{r_{\text{in}}}{r_{\text{cr}}} \right).$$

$$\frac{4\pi r_{\text{g}}^2 \sigma T_0^4}{L_{\text{E}}} = \frac{\dot{e}}{\hat{R}^2}, \quad (2)$$

where  $\dot{e}$  ( $= \dot{E}/L_{\text{E}}$ ) is the normalized energy-outflow rate (luminosity) in the comoving frame.



**Fig. 2.** Temperature distribution at the apparent photosphere as a function of radius  $r$ . The dashed curves are the temperatures in the comoving frame, while the solid ones are those in the fixed frame of the observer at infinity. The mass-outflow rate,  $\dot{m}$ , is set to be 1000. The wind velocity,  $\beta$ , is 0.1 to 0.9 in steps of 0.1 from bottom to top curves.

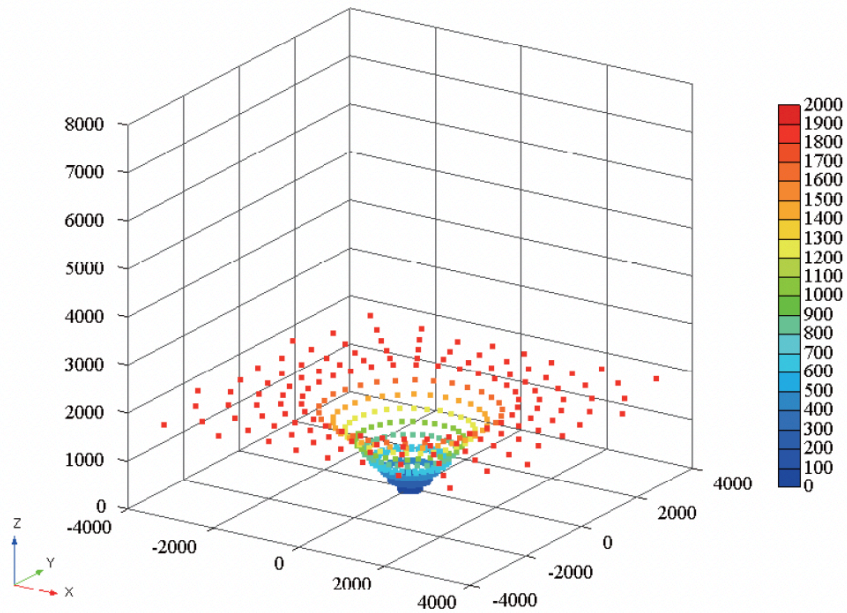
$$T_{\text{obs}} = \frac{1}{1+z} T_0 = \frac{1}{\gamma(1-\beta \cos \theta)} T_0,$$

$$L = \int_0^{r_{\text{out}}} \sigma T_{\text{obs}}^4 2\pi r dr,$$

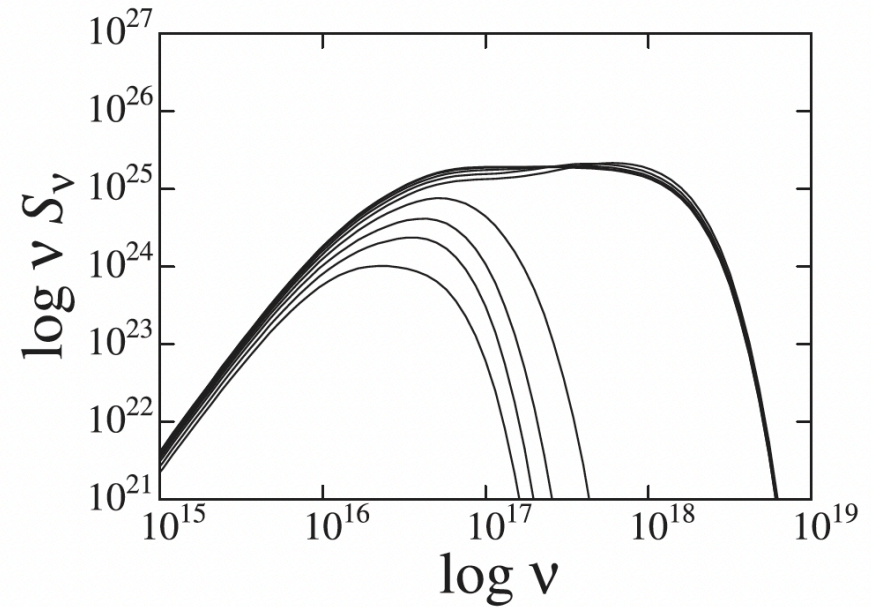
$$\tau_{\text{ph}} = \int_{z_{\text{ph}}}^{\infty} \gamma(1-\beta \cos \theta) \kappa_0 \rho_0 ds = 1,$$



# Super winds



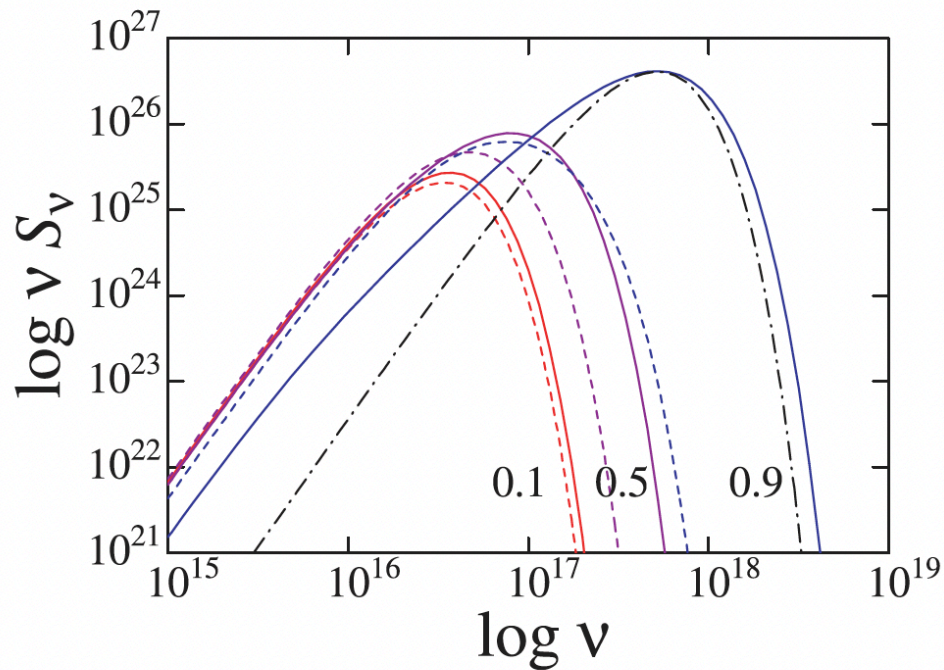
**Fig. 4.** Shape of the critical accretion disk. The inclination angle,  $i$ , is  $0^\circ$ . The mass-accretion rate is  $\dot{m}_{\text{input}} = 1000$ , and therefore,  $r_{\text{cr}} = 2000r_g$ . The axes are in units of  $r_g$ .



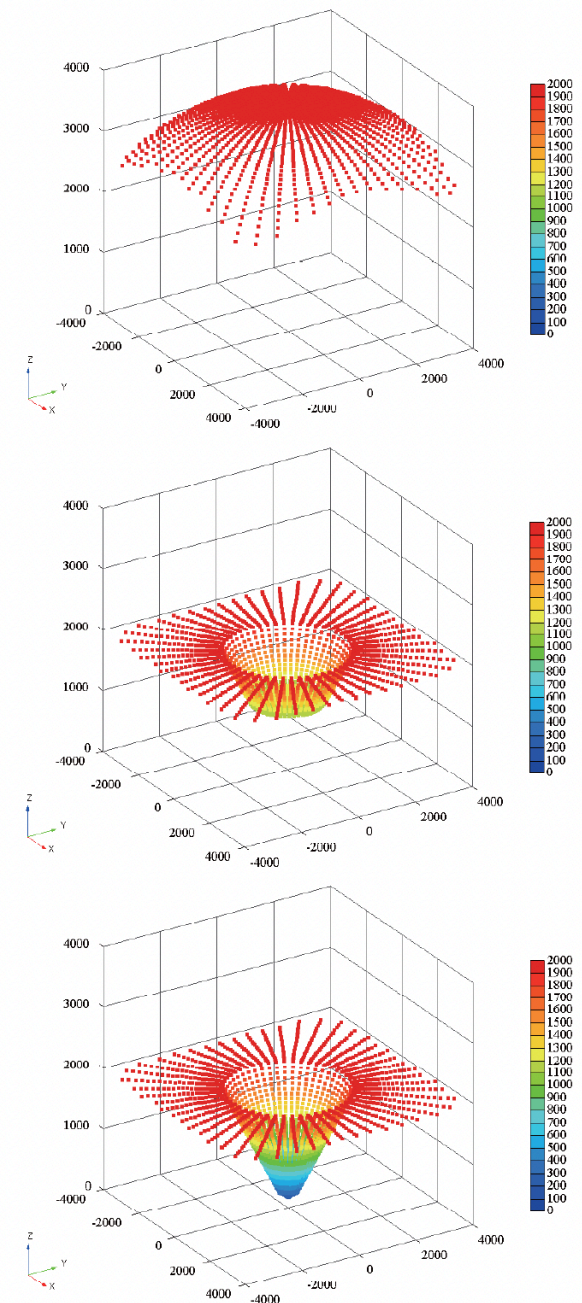
**Fig. 5.** Spectra of a naked supercritical disk. The central mass is  $m = 10$ , and the mass-accretion rate is  $\dot{m}_{\text{input}} = 1000$ . The inclination angles are  $0^\circ$  to  $80^\circ$  in steps of  $10^\circ$  from top to bottom.



# Super winds

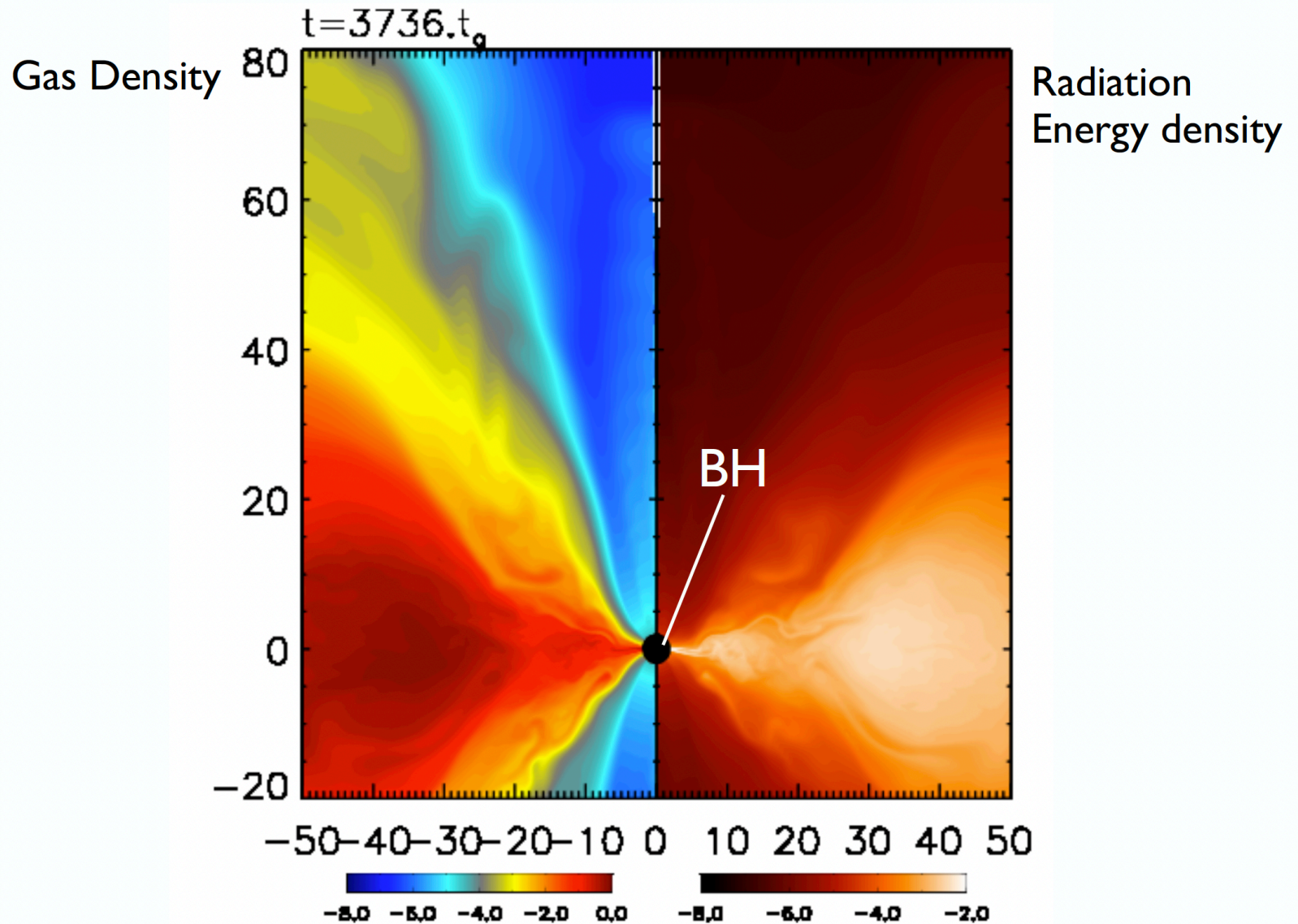


**Fig. 3.** Spectra of black hole winds. The dashed curves are the comoving spectra, while the solid ones are the observed ones. The mass-outflow rate,  $\dot{m}$ , is set to be 1000. The wind velocity,  $\beta$ , is 0.1, 0.5, and 0.9 from left to right. The chain-dotted curve means a blackbody at  $\sim 10^7$  K.



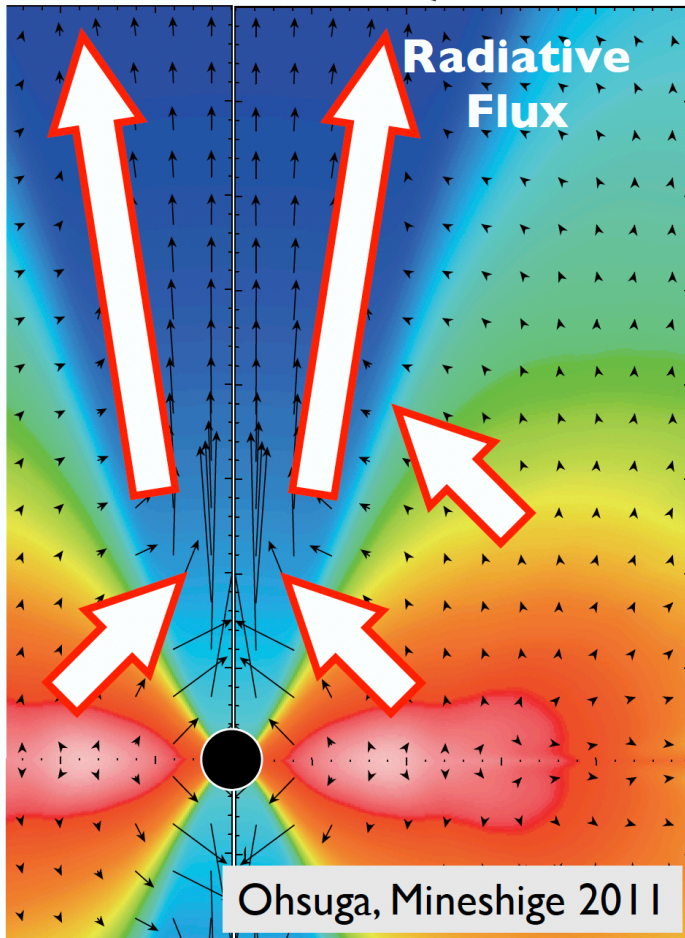
**Fig. 6.** Shape of the apparent photosphere of the black-hole wind with the supercritical disk. The inclination angle,  $i$ , is  $0^\circ$  and the wind velocity,  $\beta$ , is (a) 0.1, (b) 0.5, and (c) 0.9. The mass-outflow rate,  $\dot{m}$ , is set to be 1000. The axes are in units of  $r_g$ .

# Super-Edd. disk & Jets





# Apparent Luminosity



The radiative flux is mildly collimated since the disk is optically and geometrically thick.

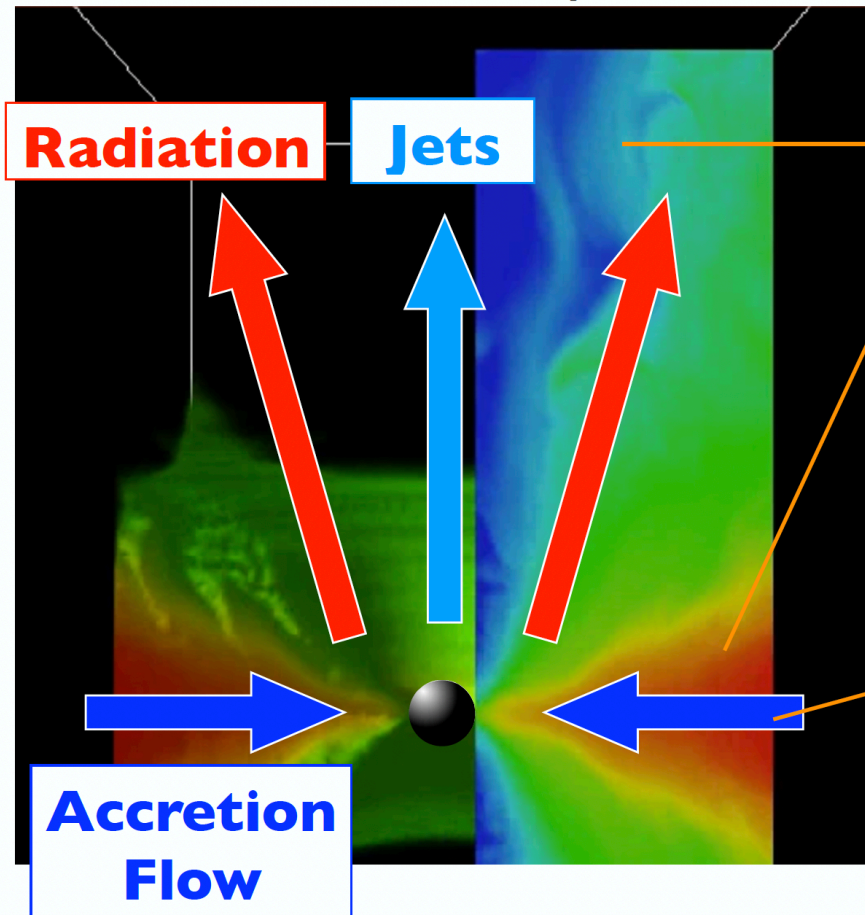
Thus observed luminosity is very sensitive to the observer's viewing angle.

The apparent luminosity becomes highly super-Eddington for the face-on observers ( $22L_{\text{Edd}}$  for  $\lesssim 20^\circ$  in the case of  $\dot{M} \sim 100L_{\text{Edd}}/c^2$ ,  $L_{\text{disk}} \sim 3L_{\text{Edd}}$ ).

Large luminosity of ULXs ( $> 10^{39-40}$  erg/s) can be explained for the face-on case.

# Super-Edd. disk & radiatively-driven jets

Mass density

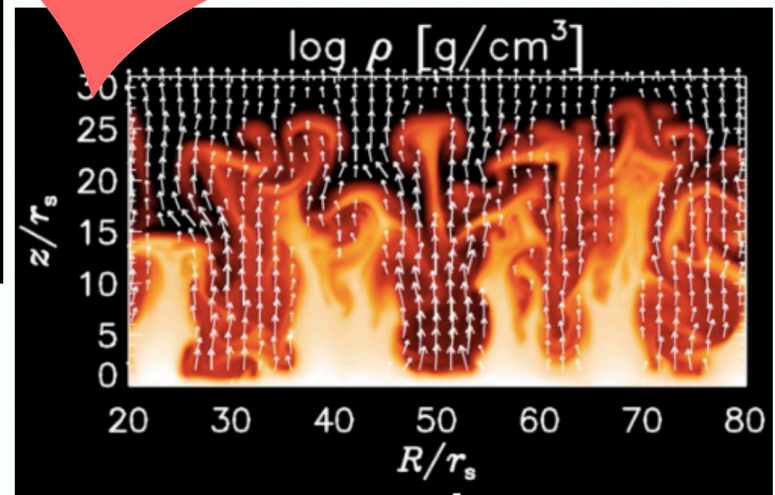
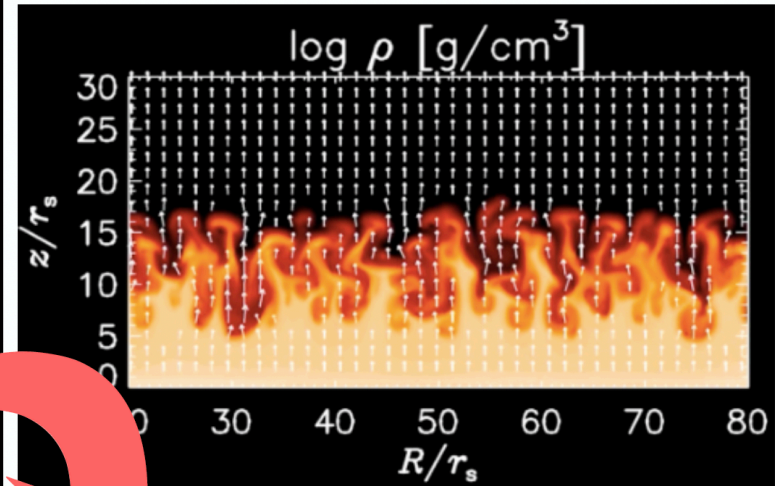
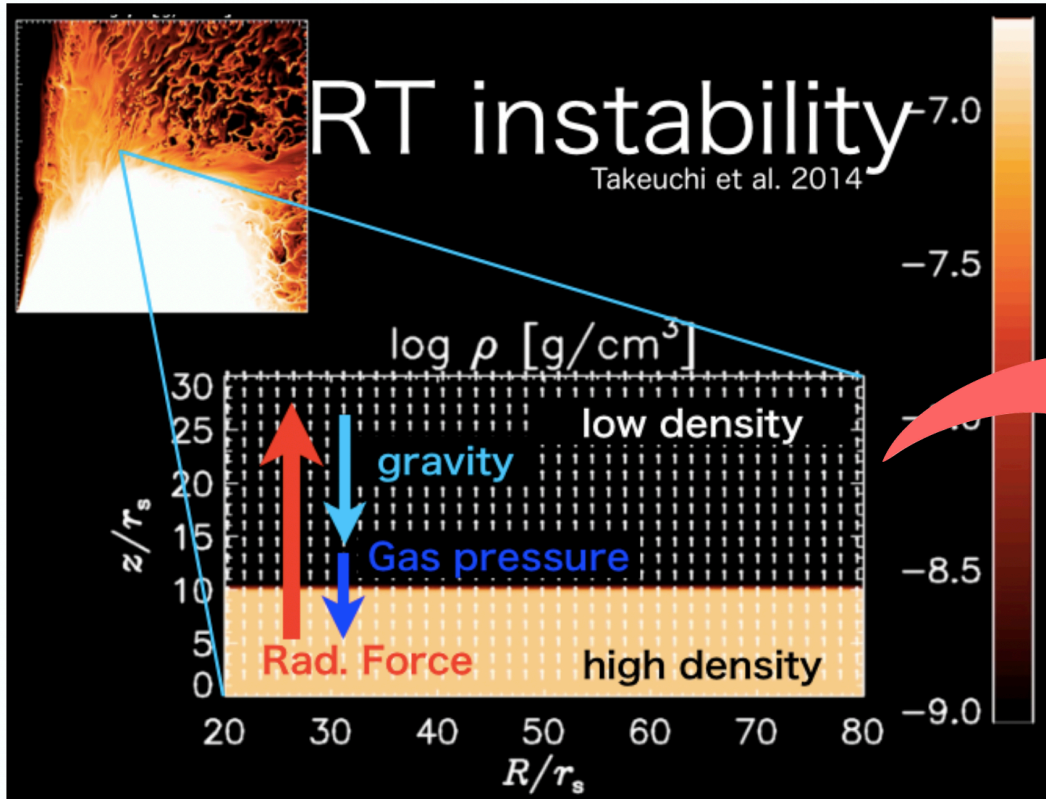


Strong radiation pressure supports the thick disk and generates the jets.

Photons mainly escape through the region around the rotation axis, so that the radiation pressure cannot prevent the accreting motion.



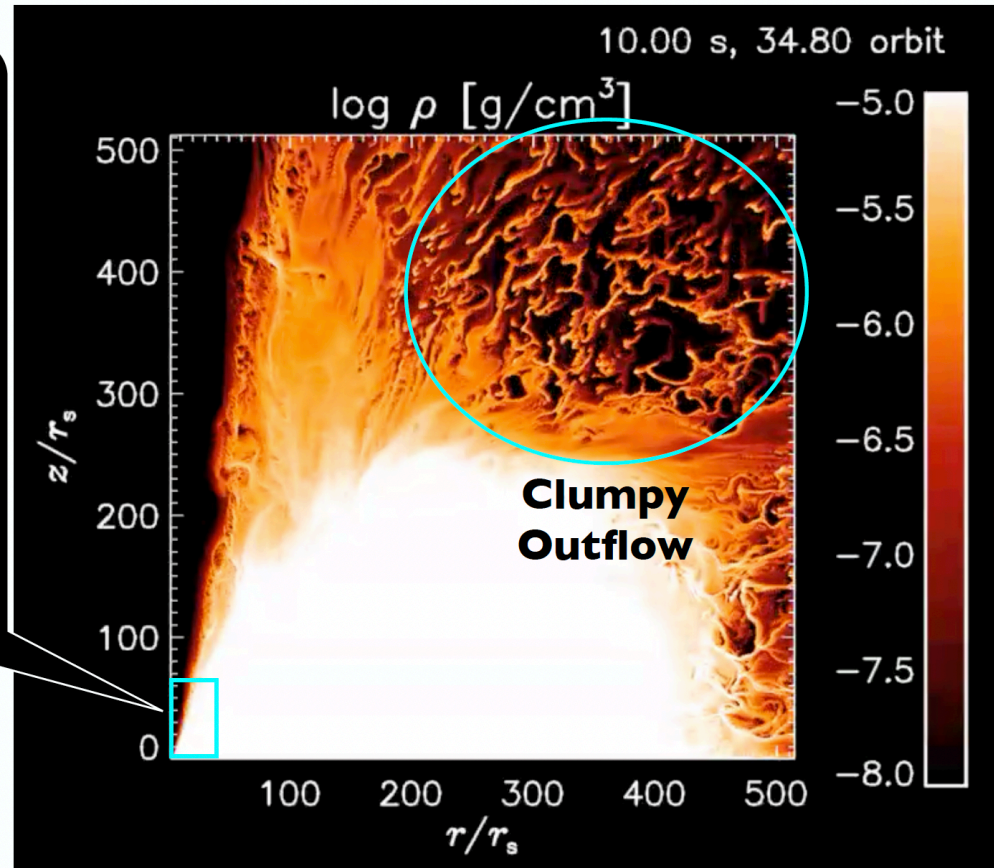
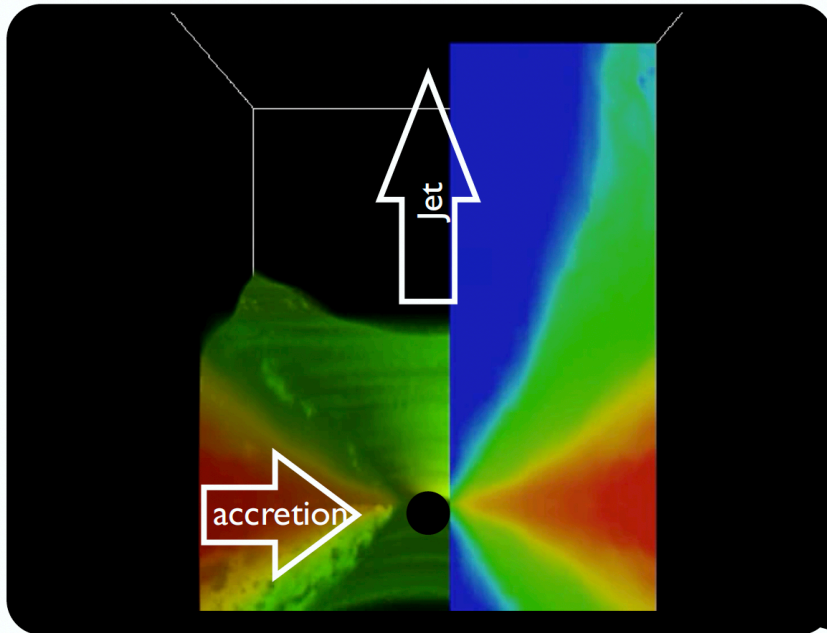
# RT instability



# Wind Outflows from Super-Edd. Disks

Takeuchi, Ohsuga, Mineshige 2013

Super-Eddington disk+ Jet



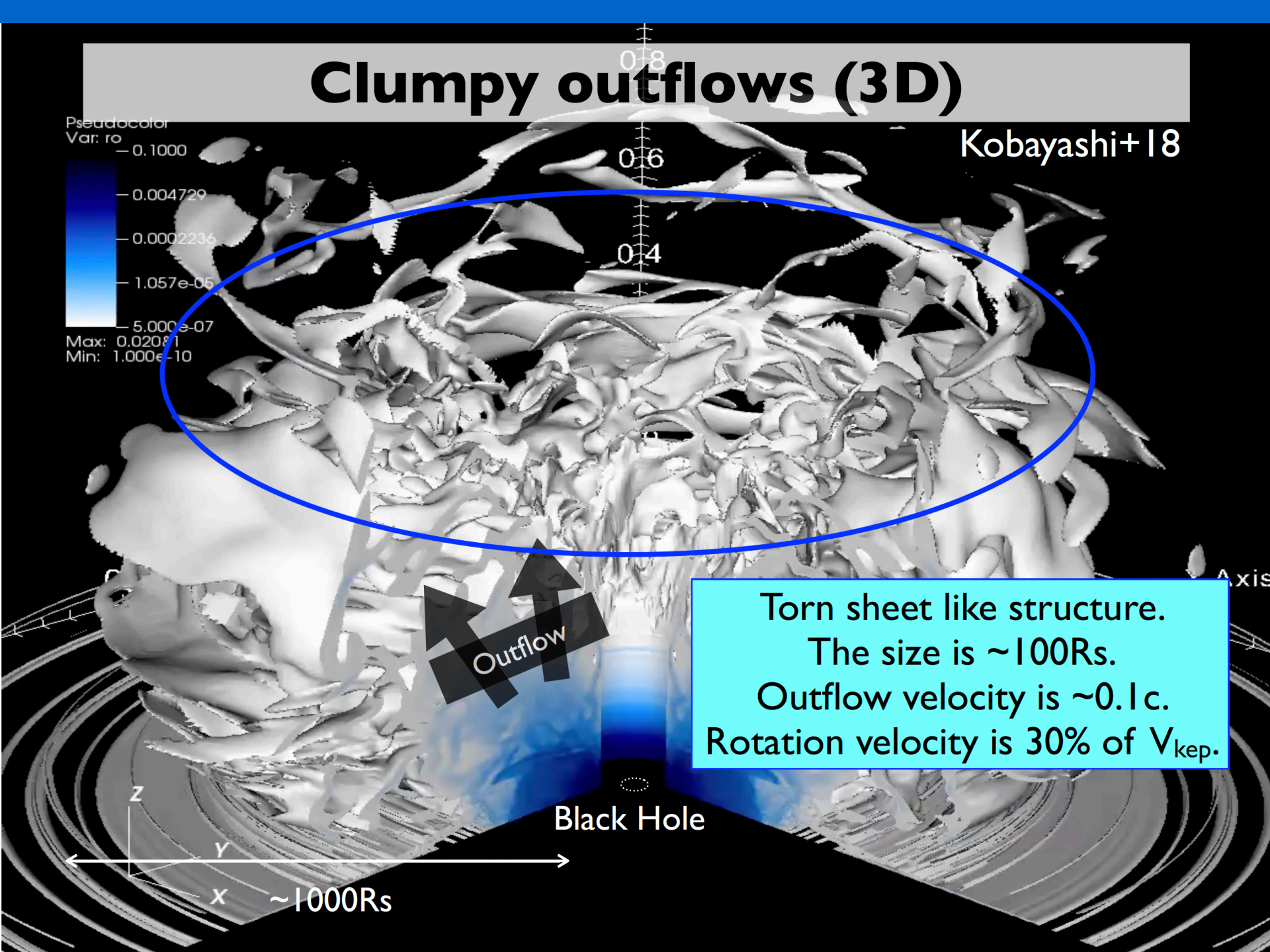
Time-dependent, Clumpy outflow  
with wide angle



# Clumpy outflows (3D)

Kobayashi+18

Pseudocolor  
Var:  $\rho_0$   
-0.1000  
-0.004729  
-0.0002236  
-1.057e-05  
-5.000e-07  
Max: 0.02081  
Min: 1.000e-10



Torn sheet like structure.  
The size is  $\sim 100R_s$ .  
Outflow velocity is  $\sim 0.1c$ .  
Rotation velocity is 30% of  $V_{\text{kep}}$ .

Black Hole

$\sim 1000R_s$



